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Solution and stability of generalized mixed type cubic, quadratic and additive functional equation in quasi-Banach spaces. (English) Zbl 1179.39034

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 71, No. 11, 5629-5643 (2009).

Let X be a real linear space. A quasi-norm is a real-valued function $\|\cdot\|$ on X satisfying the following:

- (i) $\|x\| \geq 0$ for all $x \in X$, and $\|x\| = 0$ if and only if $x = 0$;
- (ii) $\|\lambda x\| = |\lambda|\|x\|$ for all $\lambda \in \mathbb{R}$ and all $x \in X$;
- (iii) There is a constant $K \geq 1$ such that $\|x + y\| \leq K(\|x\| + \|y\|)$ for all $x, y \in X$.

Then $(X, \|\cdot\|)$ is called a quasi-normed space. A quasi-Banach space is a complete quasi-normed space. In this paper the authors investigate the generalized Hyers-Ulam-Rassias stability of the following equation

$$f(x + ky) + f(x - ky) = k^2 f(x + y) + k^2 f(x - y) + 2(1 - k^2)f(x)$$

where $k \neq 0, +1, -1$, and f is a mapping between vector spaces, and establish the generalized Hyers-Ulam-Rassias stability of the functional equation above whenever f is a function between two quasi-Banach spaces.

Reviewer: [Maryam Amyari \(Mashhad\)](#)

MSC:

- [39B82](#) Stability, separation, extension, and related topics for functional equations
- [39B52](#) Functional equations for functions with more general domains and/or ranges
- [46B99](#) Normed linear spaces and Banach spaces; Banach lattices

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Keywords:

[Hyers-Ulam-Rassias stability](#); [quadratic functional equation](#); [cubic functional equation](#); [additive functional equation](#); [quasi-Banach spaces](#)

Full Text: [DOI](#) [arXiv](#)

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