

Abbas, M.; Rhoades, B. E.

Common fixed point results for noncommuting mappings without continuity in generalized metric spaces. (English) [Zbl 1185.54037](#)

Appl. Math. Comput. 215, No. 1, 262-269 (2009).

Let $X \neq \emptyset$. Suppose that a mapping $G : X \times X \times X \rightarrow [0, \infty)$ satisfies:

- (a) $G(x, y, z) = 0$ if and only if $x = y = z$,
- (b) $0 < G(x, y, z)$ for all $x, y \in X$, with $x \neq y$.
- (c) $G(x, x, y) \leq G(x, y, z)$ for all $x, y \in X$, with $z \neq y$,
- (d) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ (symmetry in all three variables),
- (e) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then G is called a G -metric on X and (X, G) is called a G -metric space.

In the present paper the authors, using the setting of G -metric space, prove a fixed point theorem for one map, and several fixed point theorems for two maps. They prove, for example:

Theorem 2.5. Let (X, G) be a G -metric space. Suppose that $f, g : X \rightarrow X$ satisfy one of the following conditions:

$$G(fx, fy, fy) \leq k \max\{G(gx, fy, fy), G(gy, fx, fx), G(gy, fy, fy)\}$$

and

$$G(fx, fy, fy) \leq k \max\{G(gx, gx, fy), G(gy, gy, fx), G(gy, gy, fy)\}$$

for all $x, y \in X$, where $0 \leq k < 1$. If the range of g contains the range of f and $g(X)$ is a complete subspace of X , then f and g have a unique point of coincidence in X . Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Reviewer: [Jarosław Górnicki \(Rzeszów\)](#)

MSC:

54H25 Fixed-point and coincidence theorems (topological aspects)

Cited in **1** Review
Cited in **72** Documents

Keywords:

weakly compatible maps; common fixed point; generalized metric space

Full Text: [DOI](#)

References:

- [1] I. Beg, M. Abbas, Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition, Fixed Point Theor. Appl. (2006) 1-7 (Article ID 74503). · [Zbl 1133.54024](#)
- [2] Hicks, T.L.; Rhoades, B.E., A Banach type fixed point theorem, Math. japonica, 24, 3, 327-330, (1979) · [Zbl 0432.47036](#)
- [3] Jungck, G., Commuting maps and fixed points, Am. math. monthly, 83, 261-263, (1976) · [Zbl 0321.54025](#)
- [4] Jungck, G., Compatible mappings and common fixed points, Int. J. math. sci., 9, 4, 771-779, (1986) · [Zbl 0613.54029](#)
- [5] Jungck, G., Common fixed points for commuting and compatible maps on compacta, Proc. am. math. soc., 103, 977-983, (1988) · [Zbl 0661.54043](#)
- [6] Jungck, G., Common fixed points for noncontinuous nonself maps on nonmetric spaces, Far east J. math. sci., 4, 199-215, (1996) · [Zbl 0928.54043](#)
- [7] Jungck, G.; Hussain, N., Compatible maps and invariant approximations, J.m.m.a, 325, 2, 1003-1012, (2007) · [Zbl 1110.54024](#)

- [8] Z. Mustafa and B. Sims, Some Remarks concerning D -metric spaces, in: Proc. Int. Conf. on Fixed Point Theor. Appl., Valencia (Spain), July 2003, pp. 189-198. · [Zbl 1079.54017](#)
- [9] Mustafa, Z.; Sims, B., A new approach to generalized metric spaces, J. nonlinear convex anal., 7, 2, 289-297, (2006) · [Zbl 1111.54025](#)
- [10] Z. Mustafa, H. Obiedat, F. Awawdeh, Some common fixed point theorem for mapping on complete G-metric spaces, Fixed Point Theor Appl. (2008) (Article ID 189870, 12 pages). · [Zbl 1148.54336](#)
- [11] Pant, R.P., Common fixed points of noncommuting mappings, J. math. anal. appl., 188, 436-440, (1994) · [Zbl 0830.54031](#)
- [12] Park, Sehie, A unified approach to fixed points of contractive maps, J. Korean math. soc., 16, 95-105, (1980) · [Zbl 0431.54028](#)
- [13] Sessa, S., On a weak commutativity condition of mappings in fixed point consideration, Publ. inst. math. soc., 32, 149-153, (1982) · [Zbl 0523.54030](#)
- [14] Kannan, R., Some results on fixed points, Bull. Calcutta math. soc., 60, 71-76, (1968) · [Zbl 0209.27104](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.