

**Iritani, Hiroshi**

**An integral structure in quantum cohomology and mirror symmetry for toric orbifolds.**  
(English) [Zbl 1190.14054](#)  
*Adv. Math.* 222, No. 3, 1016-1079 (2009).

Consider a weak Fano projective toric orbifold  $\mathcal{X}$ . The author introduces a  $\widehat{\Gamma}$ -integral structure on the quantum  $D$ -module of  $\mathcal{X}$ , that is an integral structure on the space of flat sections of Dubrovin's connection for  $\mathcal{X}$  given by a class

$$\widehat{\Gamma}(T\mathcal{X}) = \prod_{i=1}^{\dim \mathcal{X}} \Gamma(1 + \delta_i),$$

where  $\delta_i$ 's are Chern roots of  $\mathcal{X}$ . The main theorem (Theorem 4.11) states that under some assumptions this integral structure corresponds, modulo Mirror Conjecture, to the natural integral local system on the mirror B-model  $D$ -module under the mirror isomorphism. In particular this holds for toric manifolds as assumptions are proven to hold. By assuming the existence of an integral structure, the author gives a natural explanation for the specialization to a root of unity in  $Y$ . Ruan's crepant resolution conjecture [in: AMS special session, San Francisco, CA, USA, May 3-4, 2003. Providence, RI: American Mathematical Society (AMS). *Contemporary Mathematics* 403, 117-126 (2006; [Zbl 1105.14078](#))].

Reviewer: Victor Przyjalkowski (Moskva)

**MSC:**

**14N35** Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)  
**53D45** Gromov-Witten invariants, quantum cohomology, Frobenius manifolds

Cited in **3** Reviews  
Cited in **73** Documents

**Keywords:**

quantum cohomology; variation of Hodge structures; semi-infinite variation of Hodge structures; mirror symmetry; Landau-Ginzburg model; toric Deligne-Mumford stack; orbifold; orbifold quantum cohomology; Crepant resolution conjecture; Ruan's conjecture;  $K$ -theory; McKay correspondence; oscillatory integral; hypergeometric function; GKZ-system; singularity theory; gamma class

**Full Text:** [DOI](#) [arXiv](#)

**References:**

- [1] Abouzaid, Mohammed, Morse homology, tropical geometry, and homological mirror symmetry for toric varieties · [Zbl 1204.14019](#)
- [2] Abramovich, Dan; Graber, Tom; Vistoli, Angelo, Gromov-Witten theory of Deligne-Mumford stacks, *Amer. J. math.*, 130, 5, 1337-1398, (2008), available at · [Zbl 1193.14070](#)
- [3] Adem, Alejandro; Ruan, Yongbin, Twisted orbifold  $K$ -theory, *Comm. math. phys.*, 237, 3, 533-556, (2003) · [Zbl 1051.57022](#)
- [4] Adolphson, Alan, Hypergeometric functions and rings generated by monomials, *Duke math. J.*, 73, 2, 269-290, (1994) · [Zbl 0804.33013](#)
- [5] Atiyah, Michael F.; Bott, Raoul, The moment map and equivariant cohomology, *Topology*, 23, 1, 1-28, (1984) · [Zbl 0521.58025](#)
- [6] Audin, Michele, The topology of torus actions on symplectic manifolds, *Progr. math.*, vol. 93, (1991), Birkhäuser · [Zbl 0726.57029](#)
- [7] Barannikov, Serguei, Quantum periods. I. semi-infinite variations of Hodge structures, *Int. math. res. not.*, 23, 1243-1264, (2001) · [Zbl 1074.14510](#)
- [8] Barannikov, Serguei, Semi-infinite Hodge structures and mirror symmetry for projective spaces, preprint · [Zbl 1024.32012](#)
- [9] Alexei Bondal, Wei-Dong Ruan, Mirror symmetry for weighted projective spaces, in preparation
- [10] Borisov, Lev A.; Horja, R. Paul, On the  $K$ -theory of smooth toric DM stacks, (), 21-42 · [Zbl 1171.14301](#)
- [11] Borisov, Lev A.; Horja, R. Paul, Mellin-Barnes integrals as Fourier-Mukai transforms, *Adv. math.*, 207, 2, 876-927, (2006) ·

Zbl 1137.14314

- [12] Borisov, Lev A.; Chen, Linda; Smith, Gregory G., The orbifold Chow ring of toric Deligne-Mumford stacks, *J. amer. math. soc.*, 18, 1, 193-215, (2005) · Zbl 1178.14057
- [13] Bridgeland, Tom, Stability conditions on triangulated categories, *Ann. of math. (2)*, 166, 2, 317-345, (2007) · Zbl 1137.18008
- [14] Bryan, Jim; Gholampour, Amin, Root systems and the quantum cohomology of ADE singularities, *Algebra number theory*, 2, 4, 369-390, (2008), available at · Zbl 1159.14028
- [15] Bryan, Jim; Graber, Tom, The crepant resolution conjecture, preprint · Zbl 1198.14053
- [16] Candelas, Philip; de la Ossa, Xenia C.; Green, Paul S.; Parkes, Linda, An exactly soluble superconformal theory from a mirror pair of Calabi-Yau manifolds, *Phys. lett. B*, 258, 1-2, 118-126, (1991) · Zbl 1098.32506
- [17] Cecotti, Sergio; Vafa, Cumrun, Topological-anti-topological fusion, *Nuclear phys. B*, 367, 2, 359-461, (1991) · Zbl 1136.81403
- [18] Cecotti, Sergio; Vafa, Cumrun, On classification of  $\mathcal{N} = 2\mathcal{S}$  supersymmetric theories, *Comm. math. phys.*, 158, 3, 569-644, (1993) · Zbl 0787.58049
- [19] Chen, Weimin; Ruan, Yongbin, A new cohomology theory of orbifold, *Comm. math. phys. B*, 359, 1, 1-31, (1991) · Zbl 1063.53091
- [20] Chen, Weimin; Ruan, Yongbin, Orbifold Gromov-Witten theory, (), 25-85 · Zbl 1091.53058
- [21] Coates, Tom, Wall-crossings in toric Gromov-Witten theory II: local examples, *Comm. math. phys.*, 287, 1071-1108, (2009), preprint · Zbl 1200.53081
- [22] Coates, Tom; Givental, Alexander B., Quantum Riemann-Roch, Lefschetz and Serre, *Ann. of math. (2)*, 165, 1, 15-53, (2007) · Zbl 1189.14063
- [23] Coates, Tom; Ruan, Yongbin, Quantum cohomology and crepant resolutions: A conjecture, preprint · Zbl 1275.53083
- [24] Coates, Tom; Corti, Alessio; Iritani, Hiroshi; Tseng, Hsian-Hua, Computing genus-zero twisted Gromov-Witten invariants, *Duke math. J.*, 147, 3, (2009), available at · Zbl 1176.14009
- [25] Tom Coates, Alessio Corti, Hiroshi Iritani, Hsian-Hua Tseng, Quantum cohomology of toric stacks, in preparation
- [26] Coates, Tom; Corti, Alessio; Lee, Yuan-Pin; Tseng, Hsian-Hua, The quantum orbifold cohomology of weighted projective spaces, *Acta Math.*, in press, available at · Zbl 1213.53106
- [27] Coates, Tom; Iritani, Hiroshi; Tseng, Hsian-Hua, Wall-crossings in toric Gromov-Witten theory I: crepant examples, *Geom. Topol.*, in press, available at · Zbl 1184.53086
- [28] Cox, David A.; Katz, Sheldon, Mirror symmetry and algebraic geometry, *Math. surveys monogr.*, vol. 68, (1999), Amer. Math. Soc. Providence, RI · Zbl 0951.14026
- [29] Deligne, Pierre, Local behavior of Hodge structures at infinity, (), 683-699 · Zbl 0939.14005
- [30] Douai, Antoine; Sabbah, Claude, Gauss-Manin systems, Brieskorn lattices and Frobenius structures, *Ann. inst. Fourier (Grenoble)*, 53, 1055-1116, (2003) · Zbl 1079.32016
- [31] Douglas, M.R., Dirichlet branes, homological mirror symmetry, and stability, (), 395-408 · Zbl 1008.81074
- [32] Boris Dubrovin, Geometry and analytic theory of Frobenius manifolds, in: *Proceedings of the International Congress of Mathematicians*, vol. II, Number Extra Vol. II, Berlin, 1998, pp. 315-326 (electronic), 1998 · Zbl 0916.32018
- [33] Dubrovin, Boris, Painlevé transcendents in two-dimensional topological field theory, (), 287-412 · Zbl 1026.34095
- [34] Fang, Bohan; Liu, Chiu-Chiu Melissa; Treumann, David; Zaslow, Eric,  $\text{T}$ -duality and equivariant homological mirror symmetry for toric varieties, preprint · Zbl 1260.14049
- [35] Fulton, William, Introduction to toric varieties, () · Zbl 0813.14039
- [36] Gelfand, I.M.; Zelevinsky, A.V.; Kapranov, M.M., Hypergeometric functions and toral manifolds, *Funktional. anal. i prilozhen., Funct. anal. appl.*, 23, 2, 94-106, (1989), (in Russian), translation in · Zbl 0721.33006
- [37] Givental, Alexander B., Homological geometry and mirror symmetry, (), 472-480 · Zbl 0863.14021
- [38] Givental, Alexander B., A mirror theorem for toric complete intersections, (), 141-175 · Zbl 0936.14031
- [39] Givental, Alexander B., Gromov-Witten invariants and quantization of quadratic Hamiltonians, *Mosc. math. J.*, 1, 4, 551-568, (2001), 645 · Zbl 1008.53072
- [40] Guzzetti, Davide, Stokes matrices and monodromy of the quantum cohomology of projective spaces, *Comm. math. phys.*, 207, 2, 341-383, (1999) · Zbl 0976.53094
- [41] Hertling, Claus,  $t^*$ -geometry, Frobenius manifolds, their connections, and the construction for singularities, *J. reine angew. math.*, 555, 77-161, (2003) · Zbl 1040.53095
- [42] Hertling, Claus; Sevenheck, Christian, Nilpotent orbits of a generalization of Hodge structures, *J. reine angew. math.*, 609, 23-80, (2007), available at · Zbl 1136.32011
- [43] Hori, Kentaro; Vafa, Cumrun, Mirror symmetry, preprint · Zbl 1044.14018
- [44] Horja, Richard Paul, Hypergeometric functions and mirror symmetry in toric varieties, preprint
- [45] Hosono, Shinobu, Central charges, symplectic forms, and hypergeometric series in local mirror symmetry, (), 405-439 · Zbl 1114.14025
- [46] Iritani, Hiroshi, Quantum  $\text{D}$ -modules and equivariant Floer theory for free loop spaces, *Math. Z.*, 252, 3, 577-622, (2006) · Zbl 1121.53062

- [47] Iritani, Hiroshi, Convergence of quantum cohomology by quantum Lefschetz, *J. reine angew. math.*, 610, 29-69, (2007), available at · [Zbl 1160.14044](#)
- [48] Iritani, Hiroshi, Quantum  $\mathcal{D}$ -modules and generalized mirror transformations, *Topology*, 47, 225-276, (2008), available at · [Zbl 1170.53071](#)
- [49] Iritani, Hiroshi, Real and integral structures in quantum cohomology I: toric orbifolds, preprint · [Zbl 1190.14054](#)
- [50] Iritani, Hiroshi, Ruan's conjecture and integral structures in quantum cohomology, preprint · [Zbl 1231.14046](#)
- [51] Iritani, Hiroshi,  $t^*$ -geometry in quantum cohomology, preprint · [Zbl 1231.14046](#)
- [52] Jiang, Yunfeng, The orbifold cohomology ring of simplicial toric stack bundles, preprint · [Zbl 1231.14002](#)
- [53] Katzarkov, Ludmil; Kontsevitch, Maxim; Pantev, Tony, Hodge theoretic aspects of mirror symmetry, (), 87-174, available at · [Zbl 1206.14009](#)
- [54] Kawasaki, Tetsuro, The Riemann-Roch theorem for complex  $\mathcal{V}$ -manifolds, *Osaka J. math.*, 16, 151-159, (1979) · [Zbl 0405.32010](#)
- [55] Kawasaki, Tetsuro, The index of elliptic operators over  $\mathcal{V}$ -manifolds, *Nagoya math. J.*, 84, 135-157, (1981) · [Zbl 0437.58020](#)
- [56] Kouchnirenko, A.G., Polyèdres de Newton et nombres de Milnor, *Invent. math.*, 32, 1, 1-31, (1976) · [Zbl 0328.32007](#)
- [57] Manin, Yuri I., Frobenius manifolds, quantum cohomology and moduli spaces, *Amer. math. soc. colloq. publ.*, vol. 47, (1999), Amer. Math. Soc. Providence, RI · [Zbl 0952.14032](#)
- [58] Moerdijk, Ieke, Orbifolds as groupoids: an introduction, (), 205-222 · [Zbl 1041.58009](#)
- [59] Morrison, David R., Mirror symmetry and rational curves on quintic threefolds: A guide for mathematicians, *J. amer. math. soc.*, 6, 1, 223-247, (1993) · [Zbl 0843.14005](#)
- [60] Morrison, David R., Mathematical aspects of mirror symmetry, (), 265-327 · [Zbl 0932.14022](#)
- [61] Oda, Tadao, Convex bodies and algebraic geometry. an introduction to the theory of toric varieties, *Ergeb. math. grenzgeb.* (3), vol. 15, (1988), Springer-Verlag Berlin, translated from the Japanese · [Zbl 0628.52002](#)
- [62] Palais, Richard S., Morse theory on Hilbert manifolds, *Topology*, 2, 299-340, (1963) · [Zbl 0122.10702](#)
- [63] Pandharipande, Rahul, Rational curves on hypersurfaces (after givental), *Séminaire bourbaki*, vol. 1007/98, Astérisque, 252, 5, 307-340, (1998), (Exp. No. 848) · [Zbl 0932.14029](#)
- [64] Pham, Frédéric, La descente des cols par LES ongles de Lefschetz, avec vues sur Gauss-Manin, *Astérisque*, 130, 11-47, (1985) · [Zbl 0597.32012](#)
- [65] Pressley, Andrew; Segal, Graeme, Loop groups, *Oxford math. monogr.*, (1986), The Clarendon Press, Oxford University Press · [Zbl 0618.22011](#)
- [66] Ruan, Yongbin, The cohomology ring of crepant resolutions of orbifolds, (), 117-126 · [Zbl 1105.14078](#)
- [67] Sabbah, Claude, Hypergeometric periods for a tame polynomial, *Port. math.*, C. R. sci. Paris Sér. I, 328, 7, 603-608, (1999), a short version without proofs in: · [Zbl 0967.32028](#)
- [68] Saito, Morihiko, On the structure of Brieskorn lattices, *Ann. inst. Fourier (Grenoble)*, 39, 27-72, (1989) · [Zbl 0644.32005](#)
- [69] Strominger, Andrew; Yau, Shing-Tung; Zaslow, Eric, Mirror symmetry is  $\mathcal{T}$ -duality, *Nuclear phys. B*, 479, 243-259, (1996) · [Zbl 0896.14024](#)
- [70] Toën, Bertrand, Théorèmes de Riemann-Roch pour LES champs de Deligne-Mumford,  $\mathcal{K}$ -theory, 18, 1, 33-76, (1999) · [Zbl 0946.14004](#)
- [71] Totaro, Burt, The resolution property of schemes and stacks, *J. reine angew. math.*, 577, 1-22, (2004) · [Zbl 1077.14004](#)
- [72] Tseng, Hsian-Hua, Orbifold quantum Riemann-Roch, Lefschetz and Serre, preprint · [Zbl 1178.14058](#)
- [73] Ueda, Kazushi, Stokes matrix of the quantum cohomology of cubic surfaces, preprint · [Zbl 1088.53060](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.