Fang, Chung; **Wang, Yongqi**; **Hutter, Kolumban**

A unified evolution equation for the Cauchy stress tensor of an isotropic elasto-visco-plastic material. I: On thermodynamically consistent evolution. (English) Zbl 1170.74309

Contin. Mech. Thermodyn. 19, No. 7, 423-440 (2008).

Summary: In the present study an evolution equation for the Cauchy stress tensor is proposed for an isotropic elasto-visco-plastic continuum. The proposed stress model takes effects of elasticity, viscosity and plasticity of the material simultaneously into account. It is ascribed with some scalar coefficient functions and, in particular, with an unspecified tensor-valued function N, which is handled as an independent constitutive quantity. It is demonstrated that by varying the values and the specific functional forms of these coefficients and N, different known models in non-Newtonian rheology can be reproduced. A thermodynamic analysis, based on the Müller-Liu entropy principle, is performed. The results show that these coefficients and N are not allowed to vary arbitrarily, but should satisfy certain restrictions. Simple postulates are made to further simplify the deduced general results of the thermodynamic analysis. They yield justification and thermodynamic consistency of the existing models for a class of materials embracing thermoelasticity, hypoelasticity and in particular hypoplasticity, of which the thermodynamic foundation is established successively for the first time in literature. The study points at the wide applicability and practical usefulness of the present model in different fields from non-Newtonian fluid to solid mechanics. In this paper the thermodynamic analysis of the proposed evolution-type stress model is discussed, its applications are reported later.

MSC:

74C15 Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)
74A15 Thermodynamics in solid mechanics
74B20 Nonlinear elasticity

Full Text: DOI

References:

[1] Lubliner J. (1990). Plasticity Theory. Macmillian Publishing Company, New York · Zbl 0745.73006

[2] Naghdi P.M. (1990). A critical review of the state of finite plasticity. J. Appl. Math. Phys. (ZAMP) 41: 315–394 · Zbl 0712.73032 · doi:10.1007/BF00959986

[3] Fung Y.C. (1994). A First Course of Continuum Mechanics, 3rd ed. Prentice Hall, Englewood Cliffs

[4] Truesdell, C., Noll, W.: The Non-linear Theories of Mechanics. In: Flügge, S. (ed.) Handbuch der Physik, III/3. Springer, Berlin (1965) · Zbl 0779.73004

[5] Wang Y. (2006). Time-dependent Poiseuille flows of visco-elasto-plastic fluids. Acta Mech. 186: 187–201 · Zbl 1106.76011 · doi:10.1007/s00707-006-0376-x

[6] Svendsen B., Hutter K. and Laloui L. (1999). Constitutive models for granular materials including quasi-static frictional behaviour: toward a thermodynamic theory of plasticity. Continuum Mech. Thermodyn. 4: 263–275 · Zbl 0962.74014 · doi:10.1007/s001610050115

[7] Tabor D. (1993). Gases, Liquids and Solids, 3rd ed. Cambridge University Press, Cambridge

[8] Barnes H.B., Hutton J.F. and Walters K. (1989). An Introduction to Rheology. Elsevier, Amsterdam · Zbl 0729.76001

[9] Tanner I. (1992). Engineering Rheology. Oxford University Press, New York · Zbl 1012.76002

[10] Kolymbas D. (1991). An outline of hypoplasticity. Arch. Appl. Mech. 61: 143–151 · Zbl 0734.73023

[11] Kolymbas, D.: A generalized hypoplastic constitutive law. In: Proceedings of 11th international conference on soil mechanics and foundation engineering, Vol. 5. Balkma, 2626, 1985 (1988)

[12] Kolymbas, D.: Eine konstitutive Theorie für Boden und andere körnige Stoffe. Publication Series of the Institute of Soil Mechanics and Rock Mechanics. Karlsruhe University, No. 109 (1988)

[13] Wu, W.: Hypoplasticity as a mathematical model for the mechanical behaviour of granular materials. Publication Series of the Institute of Soil Mechanics and Rock Mechanics. Karlsruhe University, No. 129 (1992)

[14] Houlsby G.T. and Puzrin A.M. (2000). An approach to plastical based on generalized thermodynamics. In: Kolymbas, D.

(eds) Constitutive Modelling of Granular Materials, pp 319–331. Springer, Berlin

[15] Houlsby, G.T.: Derivation of incremental stress–strain response for plasticity models based on thermodynamic functions. In: IUTAM symposium on mechanics of granular and porous materials. Cambridge, pp. 161–172. Kluwer, Dordrecht (1996)

[16] Collins I.F. and Houlsby G.T. (1997). Application of thermomechanical principles to the modelling of geotechnical materials. Proc. R. Soc. Lond. A 453: 1975–2001 · Zbl 0933.74045 · doi:10.1098/rspa.1997.0107

[17] Fang, C., Lee, C.H.: An unified evolution equation for the Cauchy stress tensor of an isotropic elasto-visco-plastic material. II. Normal stress difference in a viscometric flow, and an unsteady flow with a moving boundary (in press) · Zbl 1170.74308

[18] Burth K. and Brocks W. (1992). Plastizität, Grundlagen und Anwendungen für Ingenieure. Vieweg Verlag, Braunschweig

[19] Pitteri M. (1986). Continuum equations of balance in classical statistical mechanics. Arch. Ration. Mech. Anal. 94: 291–305 · Zbl 0623.73003 · doi:10.1007/BF00280907

[20] Svendsen B. (1999). A statistical mechanical formulation of continuum fields and balance relations for granular and other materials with internal degree of freedom. In: Hutter, K. and Wilmanski, K. (eds) CISM Course on Kinetic and Continuum Mechanical Approaches to Granular and Porous Media, pp 245–308. Springer, Wien · Zbl 0947.74004

[21] Müller I. (1985). Thermodynamics. Pitmann, London

[22] Hutter K. (1977). The foundations of thermodynamics, its basic postulates and implications: a review of modern thermodynamics. Acta Mech. 27: 1–54 · doi:10.1007/BF01180075

[23] Green A.E. and Naghdi P.M. (1971). On thermodynamics, rate of work and energy. Arch. Ration. Mech. Anal. 40: 38–49 · Zbl 0217.54804 · doi:10.1007/BF00281529

[24] Green A.E. and Naghdi P.M. (1972). On continuum thermodynamics. Arch. Ration. Mech. Anal. 48: 352–278 · Zbl 0255.73006 · doi:10.1007/BF00276607

[25] Liu I. (1972). Method of Lagrange multipliers for exploitation of the entropy principle. Arch. Ration. Mech. Anal. 46: 131–148 · Zbl 0252.76003

[26] Liu I. (1996). On the entropy flux-heat flux relation in thermodynamics with Lagrange multipliers. Continuum Mech. Thermodyn. 8: 247–256 · Zbl 0863.73009 · doi:10.1007/s001610050042

[27] Hutter K. and Wang Y. (2003). Phenomenological thermodynamics and entropy principle. In: Greven, A., Keller, G. and Warnecke, G. (eds) Entropy, 1st edn, pp 57–77. Princeton University Press, New Jersey · Zbl 1163.82306

[28] Wang Y. and Hutter K. (1999). A constitutive theory of fluid-saturated granular materials and its application in gravitational flows. Rheol. Acta 38: 214–223 · doi:10.1007/s003970050171

[29] Wang Y. and Hutter K. (1999). A constitutive model of multiphase mixtures and its application in shearing flows of saturated solid–fluid mixtures. Granul. Matter 1: 163–181 · doi:10.1007/s100350050023

[30] Hutter K., Laloui L. and Vulliet L. (1999). Thermodynamically based mixture models of saturated and unsaturated soils. Mech. Cohes. Frict. Mater. 4: 295–338 · doi:10.1002/(SICI)1099-1484(199907)4:4<295::AID-CFM64>3.0.CO;2-9

[31] Romano M.A. (1974). A continuum theory for granular media with a critical state. Arch. Mech. 20: 1011–1028 · Zbl 0294.73079

[32] Gudehus, G.: A comparison of some constitutive laws for soils under rapidally loading and unloading. In: Wittke, W. (ed.) Proceeding of the 3rd International Conference on Numerical Method in Geomechanics, pp. 1309–1323 (1979)

[33] Wu W. and Kolymbas D. (2000). Hypoplasticity then and now. In: Kolymbas, D. (eds) Constitutive Modelling of Granular Materials, pp 57–105. Springer, Heidelberg

[34] Luca I., Fang C. and Hutter K. (2004). A thermodynamic model of turbulent motions in a granular material. Continuum Mech. Thermodyn. 16: 363–390 · Zbl 1066.76067 · doi:10.1007/s00161-003-0163-z