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Three periodic solutions for perturbed second order Hamiltonian systems. (English)

Zbl 1185.34048
J. Math. Anal. Appl. 359, No. 2, 780-785 (2009).

Consider the following problem

$$\begin{cases} -\ddot{u} + A(t)u = \nabla F(t,u) + \lambda \nabla G(t,u) & \text{a.e. in } [0,T], \\ u(T) - u(0) = \dot{u}(T) - \dot{u}(0) = 0, \end{cases} \tag{1}$$

where $\lambda \in \mathbb{R}$, T is a positive real number. $A : [0,T] \to \mathbb{R}^{N\times N}$ is a continuous map from the interval $[0,T]$ to the set of N-order symmetric matrices, $F, G : [0,T] \times \mathbb{R}^N \to \mathbb{R}$ are measurable with respect to t for every $x \in \mathbb{R}^N$, continuously differentiable in x for almost every $t \in [0,T]$ and satisfy the following condition:

$$\sup_{|x|\le c} \max\{|F(.,x)|, |G(.,x)|, |\nabla F(.,x)|, |\nabla G(.,x)|\} \in L^1[0,T]$$

for all $c \ge 0$. Assume that

(F_1) $\lim_{|x|\to+\infty} \left(\frac{1}{2}\lambda_1(A)|x|^2 - F(t,x)\right) = +\infty$ uniformly in [0,T], where

$$\lambda_1(A) = \inf_{u\in H^1_T, \|u\|=1} \left(\int_0^T \left(|\dot{u}(t)|^2 + (A(t)u(t), u(t))\right) dt \right);$$

(F_2) there exists $\delta > 0$ such that $\frac{1}{2}\lambda_1(A)|x|^2 - F(t,x) > 0$ for all $x \in \mathbb{R}^N \backslash \{0\}$ with $|x| < \delta$ and a.e. $t \in$ [0,T];

(F_3) There exists $x_0 \in \mathbb{R}^N$ such that $\int_0^T (A(t)x_0, x_0)dt < \int_0^T F(t,x_0)dt$.

Then the authors prove that there exist $\lambda^* > 0$ and $r > 0$ such that, for every $\lambda \in]-\lambda^*, \lambda^*[$, problem (1) admits at least three distinct solutions which belong to $B_r(0) \subseteq H^1_T$ by using variational methods.

Reviewer: Chun-Lei Tang (Chongqing)

MSC:

34C25 Periodic solutions to ordinary differential equations
47J30 Variational methods involving nonlinear operators

Keywords:

second order Hamiltonian systems; periodic solutions; critical point

Full Text: DOI

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