Roulleau, Xavier

Elliptic curve configurations on Fano surfaces. (English) Zbl 1177.14079

Manuscr. Math. 129, No. 3, 381-399 (2009).

A Fano surface is the Hilbert scheme of lines of a smooth cubic threefold of $\mathbb{P}^4$. It is a smooth surface of general type with irregularity 5 and globally generated cotangent sheaf.

The author studied [Geom. Dedicata 142, 151–171 (2009; Zbl 1180.14041)] the cotangent map of these surfaces. In this paper he investigates the elliptic curves contained in a Fano surface.

The elliptic curves in a surface of general type are an obstruction for the ampleness of the cotangent sheaf, and in the case of Fano surfaces these curves are proved here to be the only obstruction. All the possible configurations of such curves are classified, i.e. their intersection matrix and a plane model of any of them are determined. Moreover, the author constructs a subgroup of the automorphism group of the surface which classifies completely these configurations.

The number of such curves is related to the Picard number. These results, as the author notes in the Introduction, imply that the ampleness of the cotangent bundle, and the geometric properties of the cotangent map, vary non-trivially in the moduli space of Fano surfaces.

Eventually the author applies is knowledge of Fano surfaces to construct a cubic threefold whose intermediate Jacobian is isomorphic – as a polarized abelian variety – to a product of elliptic curves.

Reviewer: Lidia Stoppino (Univ. dell'Insubria)

MSC:

14J29	Surfaces of general type
14J45	Fano varieties
14J50	Automorphisms of surfaces and higher-dimensional varieties
14J70	Hypersurfaces and algebraic geometry
32G20	Period matrices, variation of Hodge structure; degenerations

Keywords:

surfaces of general type; cotangent sheaf; cubic threefold

Full Text: DOI arXiv

References:

[1] Barth W., Hulek K., Peters C., Van De Ven A.: Compact complex surfaces, volume 4 of Ergebnisse der Mathematik und ihrer Grenzgebiete, 2nd edn. Springer-Verlag, Berlin (2004) · Zbl 1036.14016

[2] Beauville, A.: Les singularités du diviseur {\(\Theta\)} de la jacobienne intermédiaire de la cubique dans \({\mathbb{P}^4}\) Algebraic threefolds (Proc. Varenna 1981), LNM 947, pp. 190–208. Springer, Berlin (1982)

[3] Birkenhake, C., Lange, H.: Complex Abelian varieties, volume 302 of Grundlehren der Mathematischen Wissenschaften, 2nd edn. Springer, Berlin (2004) · Zbl 1056.14063

[4] Clemens H., Griffiths P.: The intermediate Jacobian of the cubic threefold. Ann. Math. 95, 281–356 (1972) · Zbl 0231.14004 · doi:10.2307/1970801

[5] Cohen A.: Finite complex reflection groups. Ann. Scient. Ec. Norm. Sup. 4ème Sèrie 9, 379–436 (1976) · Zbl 0359.20029

[6] Debarre, O.: Higher-dimensional Algebraic Geometry. Universitext. Springer, New York (2001) · Zbl 0978.14001

[7] Dolgachev I.: Reflection groups in algebraic geometry. Bull. Am. Math. Soc. (N.S.) 45(1), 1–60 (2008) · Zbl 1278.14001 · doi:10.1090/S0273-0979-07-01190-1

[8] Gieseker D.: On a theorem of Bogomolov on Chern classes of stables bundle. Am. J. Math. 101, 77–85 (1979) · Zbl 0431.14005 · doi:10.2307/2373939

[9] Humphreys J.: Reflection Groups and Coxeter Groups, Cambridge Studies in Advanced Mathematics, vol. 29. Cambridge University Press, Cambridge (1990) · Zbl 0725.20028

[10] Roulleau, X.: L'application cotangente des surfaces de type général. preprint Arxiv: 0902.4069, to be published in Geom.

Dedicata

[11] Shephard G.C., Todd J.A.: Finite unitary reflection groups. Can. J. Math. 5, 364–383 (1953) · Zbl 0052.16403 · doi:10.4153/CJM-1953-042-7

[12] Tyurin A.N.: On the Fano surface of a nonsingular cubic in \({\mathbb{P}^{4}}\) . Math. Ussr Izv. 4, 1207–1214 (1970) · Zbl 0225.14019 · doi:10.1070/IM1970v004n06ABEH000952

[13] Tyurin A.N.: The geometry of the Fano surface of a nonsingular cubic \({F\subset \mathbb{P}^4}\) and Torelli Theorems for Fano surfaces and cubics. Math. Ussr Izv. 5, 517–546 (1971) · Zbl 0252.14004 · doi:10.1070/IM1971v005n03ABEH001073

