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On the complex oscillation theory of $f'' + A(z)f = 0$ where $A(z)$ is analytic in the unit disc. (English) Zbl 1193.34174

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The authors study the complex oscillation theory of the second linear differential equation

$$f'' + A(z)f = 0, \tag{1}$$

where $A(z)$ is analytic in the unit disk $D = \{z : |z| < 1\}$. The order of meromorphic function f in D can be defined either as

$$\sigma(f) := \limsup_{r\to 1-} \frac{\log^+ T(r,f)}{-\log(1-r)},$$

where $T(r,f)$ is the Nevanlinna characteristic of f. Let $\lambda(f)$ denote the exponents of convergence of the sequence of zeros of the function f, $\overline{\lambda}(f)$ to denote the respectively the exponents of convergence of the sequence of distinct zeros of f, and use the notation $\sigma_2(f)$ to denote the hyper-order of $f(z)$. For analytic function f in D, we also define

$$\sigma_M(f) := \limsup_{r\to 1-} \frac{\log^+ \log^+ M(r,f)}{-\log(1-r)}.$$

The authors prove the following theorems:

Theorem 1. Let $A(z)$ be an admissible analytic function in the unit disc D. Then all nonzero solutions f of equation (1) are of infinite order and satisfy $\sigma(A) \leq \sigma_2(f) = \sigma_M(A)$.

Theorem 2. Let $A(z)$ be an admissible analytic function in the unit disc D. If $\overline{\lambda}(A) < \sigma(A)$, then every nonzero solution f of equation (1) satisfies $\sigma(A) \leq \overline{\lambda}(f)$.

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MSC:

34M10 Oscillation, growth of solutions to ordinary differential equations in the complex domain

30D35 Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

Keywords:

linear differential equations; analytic function; complex oscillation theory; exponent of convergence of zeros; exponent of convergence of distinct zeros

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