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Discretizing delta functions via finite differences and gradient normalization. (English)

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The aim of this paper is to create discretizations of certain integrals involving δ -functions in the integrand, especially in dimension n at most three. Two special types of integrals are considered, namely one that has products of δ -functions with norms of wedge products of gradients as integrands, and another one which has products of δ -functions and functions of n -variables inside the integral. For the purpose of discretizing these objects, finite difference approaches are used. Many numerical examples are offered to show the usefulness of the ideas, where of course the accuracy of the discretizations is of special interest.

Reviewer: [Martin D. Buhmann \(Gießen\)](#)

MSC:

65D15 Algorithms for approximation of functions

46F10 Operations with distributions and generalized functions

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[delta function](#); [level set method](#); [discretization](#); [high codimension](#); [finite difference](#); [approximation](#); [regular mesh](#); [numerical examples](#)

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