ten Cate, Balder; **Gabelaia, David**; **Sustretov, Dmitry**

Modal languages for topology: expressivity and definability. (English) Zbl 1172.03013

Ann. Pure Appl. Logic 159, No. 1-2, 146-170 (2009).

The topological language $\mathcal{L}_t$ [*J. Flum* and *M. Ziegler*, Topological model theory. Lecture Notes in Mathematics. 769. Berlin-Heidelberg-New York: Springer-Verlag (1980; Zbl 0421.03024)] has variables $x, y, \ldots$ for points and $U, V, \ldots$ for open sets of a topological space. Predicate symbols P_p correspond to propositional variables p. Quantifiers for sets are allowed only in combinations $\forall U(x \in U \to \alpha)$ when α is positive in U. The authors prove two characterization theorems.

Theorem. A formula $\phi(x)$ of $\mathcal{L}_t$ is equivalent to a standard translation of a propositional formula iff it is invariant under topo-bisimulations.

Theorem. A class K of topological spaces definable in $\mathcal{L}_t$ is definable in the basic modal language iff K is closed under topological sums, open subspaces and images of interior maps, while the complement of K is closed under Alexandroff extensions.

Reviewer: G. E. Mints (Stanford)

MSC:

03B45 Modal logic (including the logic of norms)
54A05 Topological spaces and generalizations (closure spaces, etc.)
03C40 Interpolation, preservation, definability

Keywords:

modal logic; topology; expressivity; definability

Full Text: DOI

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