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Cell decomposition and dimension function in the theory of closed ordered differential fields. (English) Zbl 1166.03015

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Summary: We develop a differential analogue of o-minimal cell decomposition for the theory CODF of closed ordered differential fields. Thanks to this differential cell decomposition we define a well-behaving dimension function on the class of definable sets in CODF. We conclude this paper by proving that this dimension (called δ-dimension) is closely related to both the usual differential transcendence degree and the topological dimension associated, in this case, with a natural differential topology on ordered differential fields.

MSC:

03C64 Model theory of ordered structures; o-minimality
12H05 Differential algebra
12J15 Ordered fields
12L12 Model theory of fields

Keywords:

ordered differential fields; cell decomposition; dimension function

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References:

[1] Guzy, N.; Rivière, C., Principle of differential lifting for theories of differential fields and pierce – pillay axiomatization, Notre-dame J. formal logic, 47, 331-341, (2006) · Zbl 1113.03033

[2] Hodges, W., (), xiii, 772 p

[3] Kaplansky, I., An introduction to differential algebra, (1955), Hermann

[4] Knight, J.~F.; Pillay, A.; Steinhorn, C., Definable sets in ordered structures. II, Trans. amer. math. soc., 295, 593-605, (1986) · Zbl 0662.03024

[5] L. Mathews, Topological analogues of model theoretic stability, Ph. D. Thesis, Oxford University, 1992

[6] Mathews, L., Cell decomposition and dimension functions in first-order topological structures, Proc. London math. soc., ser. III., 70, 1, 1-32, (1995) · Zbl 0829.03019

[7] C. Miller, Tameness in expansions of the real field, in: Logic Colloquium 2001, in: Lect. Notes Log. 20, Assoc. Symb. Logic, 2005, pp. 281-316 · Zbl 1081.03037

[8] Pillay, A.; Steinhorn, C., Definable sets in ordered structures. I, Trans. amer. math. soc., 295, 565-592, (1986) · Zbl 0662.03023

[9] Pillay, A.; Steinhorn, C., Definable sets in ordered structures. III, Trans. amer. math. soc., 309, 2, 469-476, (1988) · Zbl 0707.03024

[10] Rivière, C., Further notes on cell decomposition in closed ordered differential fields, Ann. pure appl. logic., 159, 1-2, 100-110, (2009) · Zbl 1166.03017

[11] C. Rivière, Model companion of theories of differential fields, Differential cell decomposition in closed ordered differential fields, Ph.D. Thesis, University of Mons-Hainaut, 2005

[12] Robinson, A., Ordered differential fields, J. combin. theory, ser. A, 14, 324-333, (1973) · Zbl 0257.12106

[13] Rolin, J.-P.; Speissegger, P.; Wilkie, A.J., Quasianalytic denjoy – carleman classes and o-minimality, J. amer. math. soc., 16, 4, 751-777, (2003) · Zbl 1095.26018

[14] Singer, M.~F., The model theory of ordered differential fields, J. symbolic logic, 43, 82-91, (1978) · Zbl 0396.03031

[15] Speissegger, P., The Pfaffian closure of an o-minimal structure, J. reine angew. math., 508, 189-211, (1999) · Zbl 1067.14519

[16] van~den Dries, L., Dimension of definable sets, algebraic boundedness and Henselian fields, Ann. pure appl. logic, 45, 2, 189-209, (1989) · Zbl 0704.03017

[17] van~den Dries, L., ()

[18] van~den Dries, L.; Speissegger, P., The real field with convergent generalized power series, Trans. amer. math. soc., 350, 11, 4377-4421, (1998) · Zbl 0905.03022

[19] van~den Dries, L.; Speissegger, P., The field of reals with multisummable series and the exponential function, Proc. London math. soc., ser. III., 81, 3, 513-565, (2000) · Zbl 1062.03029

[20] Wilkie, A., Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function, J. amer. math. soc., 9, 4, 1051-1094, (1996) · Zbl 0892.03013