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Periodic points, linearizing maps, and the dynamical Mordell-Lang problem. (English)

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J. Number Theory 129, No. 6, 1392-1403 (2009).

The paper begin by presenting the following
Question: Suppose that X is a quasi-projective variety over $\mathbb{C}$ and $\varphi : X \to X$ is a morphism. Let V be a closed subvariety of X and $\alpha \in X(\mathbb{C})$. Are there infinitely many $m \geq 0$ such that $\varphi^m(\alpha) \in V(\mathbb{C})$? Are there infinitely many of the form $kM + l$ for $M > 1$ and $l \geq 0$?

The motivation for the question is the positive answer when we have $\varphi^l(\alpha) \in W(\mathbb{C})$ for some periodic subvariety $W \subset V$. One way to attack the question is to prove that $V \cap \mathcal{O}_\varphi(\alpha)$ is at most a finite union of orbits of the form $\mathcal{O}_{\varphi^M}(\varphi^l(\alpha))$ for some M, l. The following dynamical version of the Mordell-Lang conjecture is proposed in the paper:
Conjecture: Let X be a quasi-projective variety defined over $\mathbb{C}$, $\varphi : X \to X$ be a morphism and $\alpha \in X(\mathbb{C})$, then for any subvariety $V \subset X$, the intersection $V \cap \mathcal{O}_\varphi(\alpha)$ is the union of at most finitely many orbits of the form $\mathcal{O}_{\varphi^M}(\varphi^l(\alpha))$ for some M and l.
Under suitable hypotheses the conjecture is proved for quasiprojective varieties defined over number fields and $\mathbb{C}_p$. The conjecture is also proved in the case of $X = A$ a semi-Abelian variety defined over a finitely generated subfield $K \subset \mathbb{C}$ and $\varphi : A \to A$ defined over K.
The technique of proof is as follows: Let $M > 0$ be an integer and suppose that β is a periodic point of period dividing M, the work of *M. Herman* and *J.-C. Yoccoz* [in: Geometric Dynamics, Lect. Notes Math. 1007, 408–447 (1983; Zbl 0528.58031)] provides, for any iterate $\varphi^l(\alpha)$ that is close to β, a function h on a neighborhood of β, which is p-adic analytic for a suitable p and $\varphi^M \circ h = h \circ A$ for some linear function A. When A is a homothety they apply similar techniques as used by Skolem, Mahler and Lech for linear recurrences. Based on the fact that a non-zero convergent p-adic series has at most finitely many zeros, we get that for each congruency class $i = 0, ..., M-1$ module M, either there are finitely many $n \equiv i($ mod $M)$ such that $\varphi^n(\alpha) \in V$ or we have $\varphi^n(\alpha) \in V$ for all $n \geq l$ such that $n \equiv i \pmod{M}$.

Reviewer: Jorge Pineiro (Bronx)

MSC:

14K12	Subvarieties of abelian varieties
37P35	Arithmetic properties of periodic points
37P20	Dynamical systems over non-Archimedean local ground fields
14C25	Algebraic cycles

Keywords:
subvarieties; algebraic dynamical systems; p-adic power series; linearization of p-adic maps; diagonalizable Jacobians

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