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Geometric intersection number and analogues of the curve complex for free groups. (English)

Zbl 1194.20046
Geom. Topol. 13, No. 3, 1805-1833 (2009).

From the authors' summary: "For the free group F_N of finite rank $N \geq 2$ we construct a canonical Bonahon-type, continuous and $\mathrm{Out}(F_N)$-invariant geometric intersection form $\langle\,,\,\rangle\colon \overline{\mathrm{cv}}(F_N)\times \mathrm{Curr}(F_N) \to \mathbb{R}_{\geq 0}$. Here $\overline{\mathrm{cv}}(F_N)$ is the closure of unprojectivized Culler-Vogtmann Outer space $\mathrm{cv}(F_N)$ in the equivariant Gromov-Hausdorff convergence topology (or, equivalently, in the length function topology). It is known that $\overline{\mathrm{cv}}(F_N)$ consists of all very small minimal isometric actions of F_N on $\mathbb{R}$-trees. The projectivization of $\overline{\mathrm{cv}}(F_N)$ provides a free group analogue of Thurston's compactification of Teichmüller space."

To be precise the authors prove: Let $N \geq 2$. There exists a unique continuous map $\langle\,,\,\rangle\colon \overline{\mathrm{cv}}(F_N) \times \mathrm{Curr}(F_N) \to \mathbb{R}_{\geq 0}$ which is $\mathbb{R}_{\geq 0}$-homogeneous in the first argument, $\mathbb{R}_{\geq 0}$-linear in the second argument, $\mathrm{Out}(F_N)$-invariant, and such that for every $T \in \overline{\mathrm{cv}}(F_N)$ and in every $g \in F_N \setminus \{1\}$ we have $\langle T, \eta_g\rangle = \|g\|_T$.

Let $N \geq 3$. Then the graphs $\mathcal{T}_0(F_N)$, $\mathcal{F}(F_N)$, $\mathcal{F}^*(F_N)$ have infinite diameter.

Particular interest present sections 7 and 8.

The reference list contains 38 items.

Reviewer: Stylianos Andreadakis (Athens)

MSC:

20F65 Geometric group theory
20E05 Free nonabelian groups
37E25 Dynamical systems involving maps of trees and graphs
57M07 Topological methods in group theory

Keywords:

free groups; outer space; geodesic currents; curve complexes

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