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Formal punctured ribbons and two-dimensional local fields. (English) Zbl 1168.14002

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In the classical 1-dimensional case there is a one-to-one correspondence between integral projective curves over a field k with a torsion free sheaf obeying some geometric properties and Schur pairs, i.e. pairs of k-subspaces (W, A) of $V = k((z))$ satisfying a Fredholm condition with respect to the subspace $V_+ = k[[z]]$ such that A is a k-subalgebra of V and $A \cdot W \subset W$. This is the Krichever correspondence, and in this article the authors obtain a generalization of the Krichever map for algebraic surfaces.

Parshin and Osipov established the Krichever correspondence in higher dimensions. In the 2-dimensional case it starts with a flag $(X \supset V \supset p)$, a vector bundle $\mathcal{F}$ of rank r on X, a formal trivialization e_p of $\mathcal{F}$ at p, and formal local parameters u, t at p. By these data this correspondence associates the k-subalgebra A of $V = k((u))((t))$ and k-subspace W of $V^{\oplus r}$ with Fredholm condition for all i.

Contrary to the 1-dimensional case not all such pairs of subspaces comes from geometric data. The authors solve this by introducing another type of geometric object called ribbons. The Krichever map is then decomposed into maps

$$\left\{ \begin{matrix} \text{geometric data} \\ (X, C, p, \mathcal{F}, e_p, u, t) \end{matrix} \right\} \subset \left\{ \begin{matrix} \text{geometric data} \\ \text{on ribbons} \end{matrix} \right\} \mapsto \left\{ \begin{matrix} \text{pairs of subspaces } (W, A)) \\ \text{with Fredholm condition} \end{matrix} \right\}.$$

Ribbons are ringed spaces which are more general than the notion of formal schemes, having some extra features. The authors give a thorough definition of the category of ribbons and proves the necessary properties of these geometrical objects. They also studies sheaves on ribbons and their cohomology called ind-pro-quasicoherent sheaves on ribbons, and they study their coherence property.

The Picard group of a ribbon is studied and its properties are investigated, with interesting results helping to prove the above correspondece. Also a lot of examples is given all the way.

The article is very well written, mostly self contained and with explicit constructions and examples.

Reviewer: Arvid Siqveland (Kongsberg)

MSC:

14A15 Schemes and morphisms
14A20 Generalizations (algebraic spaces, stacks)
14D05 Structure of families (Picard-Lefschetz, monodromy, etc.)

Keywords:

Krichever correspondence; Fredholm condition; ribbon; formal ribbons; ML-condition; Mittag-Leffler; function of order; Schur pair

Full Text: DOI arXiv

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