Borwein, J.; **Guirao, A. J.**; **Hájek, P.**; **Vanderwerff, J.**

Uniformly convex functions on Banach spaces. (English) Zbl 1184.52009

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Authors' summary: Given a Banach space $(X, \|\cdot\|)$, we study the connection between uniformly convex functions $f : X \to \mathbb{R}$ bounded above by $\|\cdot\|^p$ and the existence of norms on X with moduli of convexity of power type. In particular, we show that there exists a uniformly convex function $f : X \to \mathbb{R}$ bounded above by $\|\cdot\|^2$ if and only if X admits an equivalent norm with modulus of convexity of power type 2.

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MSC:

52A41 Convex functions and convex programs in convex geometry
46G05 Derivatives of functions in infinite-dimensional spaces
46N10 Applications of functional analysis in optimization, convex analysis, mathematical programming, economics
49J50 Fréchet and Gateaux differentiability in optimization
90C25 Convex programming

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convex function; uniformly smooth; uniformly convex; superreflexive

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