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The Rossby wave extra invariant in the physical space. (English) Zbl 1160.37392

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Summary: It was found out in 1991 that the Fourier space dynamics of Rossby waves possesses an extra positive-definite quadratic invariant, in addition to the energy and enstrophy. This invariant is similar to the adiabatic invariants in the theory of dynamical systems. For many years, it was unclear if this invariant – known only in the Fourier representation – is physically meaningful at all, and if it is, in what sense it is conserved. Does the extra conservation hold only for a class of solutions satisfying certain constraints (like the conservation in the Kadomtsev-Petviashvili equation)? The extra invariant is especially important because this invariant (provided it is meaningful) has been connected to the formation of zonal jets (like Jupiter's stripes). In the present paper, we find an explicit expression of the extra invariant in the physical (or coordinate) space and show that the invariant is indeed physically meaningful for any fluid flow. In particular, no constraints are needed. The explicit form also enables us to note several properties of the extra invariant.

MSC:

37K05	Hamiltonian structures, symmetries, variational principles, conservation laws (MSC2010)
37N10	Dynamical systems in fluid mechanics, oceanography and meteorology
76B65	Rossby waves (MSC2010)
76B15	Water waves, gravity waves; dispersion and scattering, nonlinear interaction

Keywords:

Rossby waves; conservation; triad resonances; drift waves in plasmas

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