Mj, Mahan

Cannon-Thurston maps for pared manifolds of bounded geometry. (English) Zbl 1166.57009

Geom. Topol. 13, No. 1, 189-245 (2009).

The author uses the following terminology:

A pared manifold is a pair (M,P) where M is a 3-manifold with boundary and P is a (possibly empty) 2-dimensional submanifold with boundary of ∂M such that:

1) the fundamental group of each component of P injects into the fundamental group of M and it contains an abelian subgroup of finite index;
2) any cylinder $C:(S^1\times I,\partial(S^1\times I))\to(M,P)$ with $C_*:\pi_1(S^1\times I)\to\pi_1(M)$ injective is homotopic rel. boundary to P;
3) P contains every component of ∂M which has an abelian subgroup of finite index.

The definition is due to Thurston.

A pared manifold (M,P) is said to have incompressible boundary if each component of $M\setminus P$ is incompressible in M.

A manifold M equipped with a metric is said to have bounded geometry if in a complement of the cusps the injectivity radius is bounded below by some positive number.

The main result of the paper is the following

Theorem: Let N^h be a hyperbolic structure of bounded geometry on a pared manifold (M,P) with incompressible boundary $\partial_0 M=(\partial M-P)$ and let M_{gf} be a geometrically finite hyperbolic structure adapted to (M,P). Then the map $i:\widetilde{M_{gf}}\to\widetilde{N^h}$ between the universal covers extends continuously to a map between the Gromov boundaries $\widehat{M_{gf}}\to\widehat{N^h}$. Furthermore, the limit set of $\widetilde{M}$ is locally connected.

This extension map between the boundaries is an example of a Cannon-Thurston map. The result of this paper is one of several results by the author on similar problems of existence of Cannon-Thurston maps, written in a series of papers.

Reviewer: Athanase Papadopoulos (Strasbourg)

MSC:

57M50	General geometric structures on low-dimensional manifolds
20F67	Hyperbolic groups and nonpositively curved groups
57N16	Geometric structures on manifolds of high or arbitrary dimension

Keywords:

Cannon-Thurston map; pared manifold; hyperbolic structure; bounded geometry

Full Text: DOI arXiv

References:

[1] W Abikoff, Kleinian groups-geometrically finite and geometrically perverse, Contemp. Math. 74, Amer. Math. Soc. (1988) 1 · Zbl 0662.30043

[2] J W Anderson, B Maskit, On the local connectivity of limit set of Kleinian groups, Complex Variables Theory Appl. 31 (1996) 177 · Zbl 0869.30034

[3] M Bestvina, Geometric group theory problem list (2004) · math.utah.edu

[4] M Bestvina, M Feighn, A combination theorem for negatively curved groups, J. Differential Geom. 35 (1992) 85 · Zbl 0724.57029 · euclid:jdg/1214447806

[5] M Bestvina, M Feighn, M Handel, Laminations, trees, and irreducible automorphisms of free groups, Geom. Funct. Anal. 7 (1997) 215 · Zbl 0884.57002 · doi:10.1007/PL00001618

[6] M Bestvina, M Handel, Train tracks and automorphisms of free groups, Ann. of Math. \((2)\) 135 (1992) 1 · Zbl 0757.57004

· doi:10.2307/2946562

[7] F Bonahon, Bouts des variétés hyperboliques de dimension \(3\), Ann. of Math. \((2)\) 124 (1986) 71 · Zbl 0671.57008 · doi:10.2307/1971388

[8] B H Bowditch, Relatively hyperbolic groups, preprint, Southampton (1997) · Zbl 1259.20052

[9] B H Bowditch, A topological characterisation of hyperbolic groups, J. Amer. Math. Soc. 11 (1998) 643 · Zbl 0906.20022 · doi:10.1090/S0894-0347-98-00264-1

[10] B H Bowditch, Convergence groups and configuration spaces (editors J Cossey, C F Miller, W D Neumann, M Shapiro), de Gruyter (1999) 23 · Zbl 0952.20032

[11] B H Bowditch, Stacks of hyperbolic spaces and ends of 3 manifolds, preprint, Southampton (2002) · Zbl 1297.57044

[12] B H Bowditch, The Cannon-Thurston map for punctured-surface groups, Math. Z. 255 (2007) 35 · Zbl 1138.57020 · doi:10.1007/s00209-006-0012-4

[13] J F Brock, Iteration of mapping classes and limits of hyperbolic 3-manifolds, Invent. Math. 143 (2001) 523 · Zbl 0969.57011 · doi:10.1007/s002220000114

[14] J F Brock, R D Canary, Y N Minsky, The Classification of Kleinian surface groups II: The Ending Lamination Conjecture, preprint (2004) · Zbl 1253.57009 · doi:10.4007/annals.2012.176.1.1

[15] J W Cannon, W P Thurston, Group invariant Peano curves, Geom. Topol. 11 (2007) 1315 · Zbl 1136.57009 · doi:10.2140/gt.2007.11.1315

[16] M Coornaert, T Delzant, A Papadopoulos, Géométrie et théorie des groupes, Lecture Notes in Mathematics 1441, Springer (1990) · Zbl 0727.20018 · doi:10.1007/BFb0084913

[17] B Farb, Relatively hyperbolic groups, Geom. Funct. Anal. 8 (1998) 810 · Zbl 0985.20027 · doi:10.1007/s000390050075

[18] W J Floyd, Group completions and limit sets of Kleinian groups, Invent. Math. 57 (1980) 205 · Zbl 0428.20022 · doi:10.1007/BF01418926 · eudml:142707

[19] É Ghys, P d l Harpe, editors, Sur les groupes hyperboliques d'après Mikhael Gromov, Progress in Mathematics 83, Birkhäuser (1990) · Zbl 0731.20025

[20] M Gromov, Hyperbolic groups, Math. Sci. Res. Inst. Publ. 8, Springer (1987) 75 · Zbl 0634.20015

[21] M Gromov, Metric structures for Riemannian and non-Riemannian spaces, Progress in Mathematics 152, Birkhäuser (1999) · Zbl 0953.53002

[22] J G Hocking, G S Young, Topology, Addison-Wesley Publishing Co., Reading, MA-London (1961) · Zbl 0135.22701

[23] E Klarreich, Semiconjugacies between Kleinian group actions on the Riemann sphere, Amer. J. Math. 121 (1999) 1031 · Zbl 1011.30035 · doi:10.1353/ajm.1999.0034 · muse.jhu.edu

[24] C T McMullen, Amenability, Poincaré series and quasiconformal maps, Invent. Math. 97 (1989) 95 · Zbl 0672.30017 · doi:10.1007/BF01850656 · eudml:143694

[25] C T McMullen, Iteration on Teichmüller space, Invent. Math. 99 (1990) 425 · Zbl 0695.57012 · doi:10.1007/BF01234427 · eudml:143766

[26] C T McMullen, Local connectivity, Kleinian groups and geodesics on the blowup of the torus, Invent. Math. 146 (2001) 35 · Zbl 1061.37025 · doi:10.1007/PL00005809

[27] Y N Minsky, Teichmüller geodesics and ends of hyperbolic \(3\)-manifolds, Topology 32 (1993) 625 · Zbl 0793.58010 · doi:10.1016/0040-9383(93)90013-L

[28] Y N Minsky, On rigidity, limit sets, and end invariants of hyperbolic \(3\)-manifolds, J. Amer. Math. Soc. 7 (1994) 539 · Zbl 0808.30027 · doi:10.2307/2152785

[29] Y N Minsky, The classification of punctured-torus groups, Ann. of Math. \((2)\) 149 (1999) 559 · Zbl 0939.30034 · doi:10.2307/120976 · www.math.princeton.edu · eudml:120365

[30] Y N Minsky, Bounded geometry for Kleinian groups, Invent. Math. 146 (2001) 143 · Zbl 1061.37026 · doi:10.1007/s002220100163 · arxiv:math/0105078

[31] Y N Minsky, The classification of Kleinian surface groups I: Models and Bounds, preprint (2002) · Zbl 1193.30063 · doi:10.4007/annals.2010.171.1 · annals.princeton.edu

[32] Y N Minsky, End invariants and the classification of hyperbolic 3-manifolds, Int. Press, Somerville, MA (2003) 181 · Zbl 1049.57010

[33] M Mitra, PhD thesis, UC Berkeley (1997)

[34] M Mitra, Ending laminations for hyperbolic group extensions, Geom. Funct. Anal. 7 (1997) 379 · Zbl 0880.57001 · doi:10.1007/PL00001624

[35] M Mitra, Cannon-Thurston maps for hyperbolic group extensions, Topology 37 (1998) 527 · Zbl 0907.20038 · doi:10.1016/S0040-9383(97)00036-0

[36] M Mitra, Cannon-Thurston maps for trees of hyperbolic metric spaces, J. Differential Geom. 48 (1998) 135 · Zbl 0906.20023 · euclid:jdg/1214460609

[37] H Miyachi, Semiconjugacies between actions of topologically tame Kleinian groups, preprint (2002)

[38] M Mj, Cannon-Thurston Maps for Surface Groups I: Amalgamation Geometry and Split Geometry (2005) · arxiv:math.GT/0512539

[39] M Mj, Cannon-Thurston maps, i-bounded geometry and a theorem of McMullen (2005) · Zbl 1237.57018 · arxiv:math.GT/0511041 · eudml:116466

[40] M Mj, Cannon-Thurston Maps for Surface Groups II: Split Geometry and the Minsky Model (2006) · arxiv:math.GT0607509

[41] M Mj, A Pal, Relative Hyperbolicity, Trees of Spaces and Cannon-Thurston Maps (2007) · Zbl 1222.57013 · doi:10.1007/s10711-010-9519-2 · arxiv:0708.3578

[42] R M Myers, From Beowulf to Virginia Woolf: An astounding and wholly unauthorized history of English literature, Bobbs-Merrill, Indianapolis (1952)

[43] A Pal, Relative Hyperbolic Extensions of Groups and Cannon-Thurston Maps (2008) · arxiv:0801.0933

[44] G P Scott, Compact submanifolds of \(3\)-manifolds, J. London Math. Soc. \((2)\) 7 (1973) 246 · Zbl 0266.57001 · doi:10.1112/jlms/s2-7.2.246

[45] J Souto, Cannon-Thurston maps for thick free groups, preprint (2006)

[46] W P Thurston, The geometry and topology of 3-manifolds, Princeton University notes (1980)

[47] W P Thurston, Hyperbolic structures on \(3\)-manifolds. I. Deformation of acylindrical manifolds, Ann. of Math. \((2)\) 124 (1986) 203 · Zbl 0668.57015 · doi:10.2307/1971277

[48] W P Thurston, Hyperbolic structures on \(3\)-manifolds. III. Deformation of 3-manifolds with incompressible boundary, preprint (1986) · Zbl 0668.57015 · doi:10.2307/1971277

[49] A Yaman, A topological characterisation of relatively hyperbolic groups, J. Reine Angew. Math. 566 (2004) 41 · Zbl 1043.20020 · doi:10.1515/crll.2004.007