Downey, Peter J.; **Wright, Paul E.**

The ratio of the extreme to the sum in a random sequence. (English) Zbl 1164.60021

Extremes 10, No. 4, 249-260 (2007).

The ratio $R_n = X_{(n)}/S_n$ is considered, where $S_n = \sum_{i=1}^n X_i$, $X_{(n)} = \max_{1\le i\le n} X_i$, X_i are i.i.d. random variables. It is shown that $\mathbf{E}R_n = \frac{\mathbf{E}X(n)}{\mathbf{E}S_n}(1 + o(1))$ as $n \to \infty$ if $\mathbf{E}X_i^2 < \infty$ or if the survival function of X_i is regularly varying with the index of variation less then -1. The proof is based on an integral representation for $\mathbf{E}R_n$. The results are applied to a multiprocessor scheduling asymptotical analysis.

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MSC:

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68M20 Performance evaluation, queueing, and scheduling in the context of computer systems

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