Castryck, Wouter

A shortened classical proof of the quadratic reciprocity law. (English) Zbl 1228.11006
Am. Math. Mon. 115, No. 6, 550-551 (2008).

The author simplifies V. A. Lebesgue's proof of the quadratic reciprocity law $(p/q)(q/p) = (-1)^{(p-1)(q-1)/4}$, which was based on counting the number of solutions $x_1^2 + x_2^2 + \cdots + a_n^2 = 1$ over $\mathbb{F}_q$. In this article, the number of solutions of $x_1^2 - x_2^2 + x_3^2 - \cdots + a_n^2 = 1$ for odd integers n is easily computed by induction as $N_n = q^{n-1} + q^{(n-1)/2}$. Thus $N_p \equiv 1 + (q/p) \bmod p$ by Fermat and Euler's criterion, and invoking a simple calculation of a multiple Jacobi sum, the reciprocity law follows.

Reviewer: Franz Lemmermeyer (Jagstzell)

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