

**Birman, Joan S.; Lubotzky, Alex; McCarthy, John**

**Abelian and solvable subgroups of the mapping class group.** (English) Zbl 0551.57004  
Duke Math. J. 50, 1107-1120 (1983).

The main result of the paper is the following theorem: every solvable subgroup of the mapping class group of an orientable surface contains an abelian subgroup of finite index. This result is complemented by bounds on the rank and the index of this subgroup; the first one is precise. The proofs are based on Thurston's theory of diffeomorphisms of surfaces, to which the authors give some nice complements. The importance of the paper lies in that it indicates the effectiveness of Thurston's theory in the study of algebraic properties of the mapping class groups of surfaces. The paper has become the starting point of some further works. In particular, the above theorem was generalized by J. McCarthy and the reviewer (simultaneously and independently) to the following analogue of a famous Tits' theorem in the theory of linear groups: every subgroup of the mapping class group of an orientable surface contains either a free group with two generators or an abelian subgroup of finite index. See the third author, A "Tits alternative" for subgroups of surface mapping class groups, Preprint, 1984, and the reviewer, Dokl. Akad. Nauk SSSR 275, 786-789 (1984).

Reviewer: N.V.Ivanov

**MSC:**

57N05 Topology of the Euclidean 2-space, 2-manifolds (MSC2010)  
57R50 Differential topological aspects of diffeomorphisms  
30F99 Riemann surfaces

Cited in **5** Reviews  
Cited in **77** Documents

**Keywords:**

free nonabelian subgroups of the mapping class group; solvable subgroup of the mapping class group of an orientable surface; abelian subgroup of finite index; diffeomorphisms of surfaces

**Full Text:** [DOI](#)

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