Abidi, Hammadi

On the uniqueness for the Boussinesq system with non linear diffusion. (Sur l'unicité pour le système de Boussinesq avec diffusion non linéaire.) (French) Zbl 1156.35074

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The author considers uniqueness problem for the 2D Boussinesq system (BS) with nonlinear diffusion, in critical spaces

$$\begin{cases} \partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v} - \operatorname{div}(2\mu(\theta)\mathcal{M}) + \nabla p = 0, \\ \partial_t\theta + (\vec{v}\cdot\nabla)\theta = 0, \\ \operatorname{div}\vec{v} = 0, \\ (\vec{v},\theta)|_{t=0} = \left(\vec{v}^0,\theta^0\right), \end{cases} \tag{B}$$

where $\mathcal{M}$ is the strain tensor, $\vec{v}=(v_1,v_2)$ is the velocity, p is the pressure and the kinematic viscosity μ is a positive C^∞ function satisfying the uniform lower bound

$$0 < \underline{\mu} \leq \mu(s) \text{ for any } s > 0. \tag{visc}$$

The main result is the following: provided that $v_j^0 \in \dot{B}^{-1}_{\infty,1}(\mathbb{R}^2)\cap L^2(\mathbb{R}^2)$ for $j=1,2$, with $\vec{v}^{\,0}$ divergence-free and $\theta^0 \in \dot{B}^{1}_{2,1}(\mathbb{R}^2)$ and if μ satisfies (visc), there exists $\epsilon > 0$ small enough such that if

$$\|\mu(\theta_0)-1\|_{L^\infty} \leq \epsilon \text{ and } \|\theta^0\|_{\dot{B}^1_{2,1}(\mathbb{R}^2)} \leq \epsilon,$$

then there exists a $T(\theta^0,\vec{v}^{\,0})$ such that (B) has a unique solution $(\vec{v},\theta)$ satisfying

$$v_j \in C_b([0,T);\dot{B}^{-1}_{\infty,1}(\mathbb{R}^2)) \cap L^1([0,T);\dot{B}^{-1}_{\infty,1}(\mathbb{R}^2)) \cap L^\infty([0,T);L^2(\mathbb{R}^2)) \cap L^2([0,T);\dot{H}^1(\mathbb{R}^2)),$$

for $j=1,2$ and $\theta \in C_b([0,T);\dot{B}^1_{2,1}(\mathbb{R}^2))$.

Reviewer: Bernard Ducomet (Bruyères le Châtel)

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Boussinesq system; Uniqueness; Paradifferential calculus

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