

**Cánovas, J. S.; Linero Bas, A.; Soler López, G.**

**On closed subgroups associated with involutions.** (English) [Zbl 1254.39010](#)

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Summary: Given an involution  $f$  on  $(0, \infty)$ , we prove that the set  $\mathcal{C}(f) := \{\lambda > 0 : \lambda f \text{ is an involution}\}$  is a closed multiplicative subgroup of  $(0, \infty)$  and therefore  $\mathcal{C}(f)$  is  $\{1\}$ , or  $\lambda^{\mathbb{Z}} = \{\lambda^n : n \in \mathbb{Z}\}$  for some  $\lambda > 0, \lambda \neq 1$ . Moreover, we provide examples of involutions possessing each one of the above types as the set  $\mathcal{C}(f)$  and prove that the unique involutions  $f$  such that  $\mathcal{C}(f) = (0, \infty)$  are  $f(x) = c/x, c > 0$ .

**MSC:**

**39B22** Functional equations for real functions

**26A18** Iteration of real functions in one variable

**Keywords:**

involutions; closed subgroups; difference and functional equations

**Full Text:** [DOI](#)