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Divisor arrangements and algebraic surfaces. (English) Zbl 1158.14019

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The Chern numbers c_1^2 and c_2 of a minimal complex surface of general type satisfy the famous Bogomolov-Miyaoka-Yau inequality $c_1^2 \leq 3c_2$.

It is quite hard to construct explicitly surfaces with $c_1^2 = 3c_2$, and even surfaces with the ratio c_1^2/c_2 close to 3 are not easily found. Some examples with $c_1^2 = 3c_2$ have been given in *Friedrich Hirzebruch* [Arithmetic and geometry, Pap. dedic. I. R. Shafarevich, Vol. II: Geometry, Prog. Math. 36, 113–140 (1983; Zbl 0527.14033)]. They are minimal desingularizations of Galois covers of the plane branched on certain arrangements of lines.

In the paper under review, the author generalizes Hirzebruch's construction by defining an arrangement of curves on a smooth surface S as a set of curves $L_1, \dots L_t$ together with a set of divisors $H_0, \dots H_k$ supported on $L_1, \dots L_t$ such that some geometrical and combinatorial conditions are satisfied. It is shown that these conditions imply that for infinitely many integers n there exists a smooth Galois cover $S_n \to S$ with Galois group Z_n^k branched on $L_1, \dots L_t$ and the Chern numbers of S_n are computed.

The following example is worked out explicitly in the paper: S is the Fano surface of lines of the Fermat cubic hypersurface F defined by $x_0^2 + \dots + x_4^2 = 0$ and the L_i are 30 smooth elliptic curves corresponding to the lines contained in certain special hyperplane sections of F. In this case the construction gives a surface with $c_1^2/c_2 = 3 - 1/27$ and an infinite sequence of surfaces with c_1^2/c_2 converging to $5/2$.

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MSC:

14E20 Coverings in algebraic geometry
14J45 Fano varieties
14J29 Surfaces of general type

Keywords:

surface arrangement; Chern numbers; covering; Fano surface