Mitake, Hiroyoshi

The large-time behavior of solutions of the Cauchy-Dirichlet problem for Hamilton-Jacobi equations. (English) Zbl 1171.35332

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The large-time behavior of solutions of the Cauchy-Dirichlet problem for the Hamilton-Jacobi equation

$$\begin{cases} u_t(x,t) + H(x, Du(x,t)) = 0 & \text{in } Q, \\ u(x,t) = f(x) & \text{in } \Omega \times \{0\}, \\ u(x,t) = g(x) & \text{on } \partial\Omega \times (0,\infty) \end{cases} \tag{1}$$

is investigated. Here Ω is a bounded domain of $\mathbb{R}^n$, $Q = \Omega \times (0,\infty)$, the Hamiltonian $H = H(x,p)$ is a real-valued function on $\overline{\Omega} \times \mathbb{R}^n$ which is coercive and convex in the variable p and $f : \overline{\Omega} \to \mathbb{R}$, $g : \partial\Omega \to \mathbb{R}$ are given functions. This work deals only with the viscosity solutions of the Hamilton-Jacobi equations. In recent years, many researchers have investigated the large-time behavior of the solution $u(x,t)$ of (1) as $t \to \infty$.

In this paper, the author does not assume the compatibility condition on the initial and boundary data f, g. This gives a viewpoint which unifies the state constraint and Dirichlet boundary conditions. In the last part of the paper, general convergence results for viscosity solutions of (1) by using the Aubry-Mather theory are established and representation formulas for asymptotic solutions are given.

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MSC:

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35F25 Initial value problems for nonlinear first-order PDEs
35F30 Boundary value problems for nonlinear first-order PDEs

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