Simon, Leon

Asymptotics for a class of non-linear evolution equations, with applications to geometric problems. (English) Zbl 0549.35071

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The first part of this paper is concerned with the study of the asymptotic behaviour of a smooth function u(x,t) defined on $\Sigma\times[0,T)$ and satisfying either (1) $\dot u-\mathcal{M}(u)=f$ or (2) $\ddot u+\dot u-\mathcal{M}(u)-\mathcal{R}(u)=f$. Here Σ is a compact Riemannian manifold, f(x,t) decays exponentially for $t\to\infty$, and $\mathcal{M}(u)$ is a second order elliptic Euler-Lagrange operator associated with an energy functional $\mathcal{E}(u)=\int_\Sigma E(x,u,\nabla u)$; E depends analytically on u and ∇u for $\|u\|_{C^1(\Sigma)}$ sufficiently small and $\mathcal{M}(0)=0$. The term $\mathcal{R}(u)$ is assumed to be negligible in a certain sense. For a solution u of (1) with small initial data it is shown that either $\mathcal{E}(u(t))$ becomes negative for some t, or u(t) converges asymptotically to a solution of $\mathcal{M}(\omega)=0$.

The main result concerning the hyperbolic equation states that a solution with small initial data and not too fast growth converges to a solution of the stationary equation provided $\mathcal{E}(u(t))-\mathcal{E}(0)\geq\delta$, δ sufficiently small. Among the important applications to geometric problems are the following. The shown behaviour of the solution of (2) at infinity implies that an n-dimensional minimal submanifold of $\mathbb{R}^{n+k}(k\geq 1)$ with a singularity at 0 and which has a multiplicity 1 "tangent cone" must converge asymptotically to this cone. This improves results obtained by *W. K. Allard* and *F. J. Almgren* [ibid. 113, 215-265 (1981; Zbl 0437.53045)].

Another application shows that "tangent maps" for an energy minimizing harmonic map at an isolated singularity is unique, in case the target manifold is an analytic submanifold isometrically imbedded in an Euclidean space and in case at least one of the tangent maps are smooth.

Reviewer: R.Landes

MSC:

35L10	Second-order hyperbolic equations
49Q20	Variational problems in a geometric measure-theoretic setting
58J35	Heat and other parabolic equation methods for PDEs on manifolds
58J45	Hyperbolic equations on manifolds
58E15	Variational problems concerning extremal problems in several variables; Yang-Mills functionals
58E20	Harmonic maps, etc.
53C42	Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)

Keywords:

evolution equations; geometric problems; asymptotic behaviour; compact Riemannian manifold; small initial data

Full Text: DOI