

**Engau, Alexander**

**Variable preference modeling with ideal-symmetric convex cones.** (English) Zbl 1166.90014  
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The paper addresses the problem of modeling preferences in multiple criteria decision making and multiobjective programming. Variable domination structures, where the dominated set of any point  $y$  is modeled by an ideal-symmetric convex cone  $D(y)$  that contains the nonnegative orthant, are used for this purpose. Well known results for multicriteria optimization with constant domination structures are generalized to this case. The results include results on weighted sum scalarization, necessary and sufficient conditions for nondominated points and further results for problems where the nondominated set  $N(Y, \mathbb{R}_\geq^m)$  is  $\mathbb{R}_\geq^m$ -convex or  $N(Y, \mathbb{R}_\geq^m)$  is  $\mathbb{R}_\geq^m$ -concave and  $\mathbb{R}_\geq^m$ -compact. The paper also contains some examples.

Reviewer: [Matthias Ehrgott \(Auckland\)](#)

**MSC:**

**90C29** Multi-objective and goal programming  
**90B50** Management decision making, including multiple objectives

Cited in **18** Documents

**Keywords:**

[Multiobjective programming](#); [multicriteria optimization](#); [ordering cones](#); [preference models](#)

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