Silverman, Joseph H.

Variation of periods modulo p in arithmetic dynamics. (English) Zbl 1153.11028

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Let V be a quasi-projective variety over a number field K, and let $\varphi : V \to V$ be a K-morphism. We fix a K-rational point P on V, and assume that $O_\varphi(P) := \{\varphi^n(P) \mid n \in \mathbb{N}\}$ is an infinite set. If φ has good reduction at a finite place $\mathfrak{p}$ of K, then we set $m_\mathfrak{p}(\varphi, P) := \#\{\widetilde{Q} \mid Q \in O_\varphi(P)\}$, where $\widetilde{Q}$ is the reduction of P at $\mathfrak{p}$. Otherwise, we set $m_\mathfrak{p}(\varphi, P) := \infty$. The author proves that, roughly speaking, $m_\mathfrak{p}\varphi, P)$ is almost as large as $\log N_{K/\mathbb{Q}}\mathfrak{p}$ for most $\mathfrak{p}$. To state precise statements, given a set $\mathcal{P}$ of finite places of K, we define $\delta(\mathcal{P})$ (resp. $\underline{\delta}(\mathcal{P})$) to be the limit (resp. liminf) of $(d\log\zeta_K(\mathcal{P}, s))(d\log\zeta_K(s))^{-1}$ as $s \to 1+$, where $\zeta_K(s)$ is the Dedekind zeta function of K and $\zeta_K(\mathcal{P}, s) = \prod_{\mathfrak{p}\in\mathcal{P}}(1 - (N_{K/\mathbb{Q}}\mathfrak{p})^{-s})^{-1}$ is the partial zeta function for $\mathcal{P}$. The main results of the paper under review are the following:

(1) For any $\gamma < 1$, we have

$$\delta\{\mathfrak{p} \mid m_\mathfrak{p}(\varphi, P) \geq (\log N_{K/\mathbb{Q}}\mathfrak{p})^\gamma\} = 1.$$

(2) There exists $C = C(K, V, \varphi, P)$ so that for all $\varepsilon > 0$

$$\underline{\delta}\{\mathfrak{p} \mid m_\mathfrak{p}(\varphi, P) \geq \varepsilon \log N_{K/\mathbb{Q}}\mathfrak{p}\} \geq 1 - C\varepsilon.$$

The author expects this lower bound is far from strict. Based on experimental and heuristic arguments, he made a conjecture that for any $\varepsilon > 0$, one should have

$$\delta\{\mathfrak{p} \mid m_\mathfrak{p}(\varphi, P) \leq N_{K/\mathbb{Q}}\mathfrak{p}^{\frac{N}{2}-\varepsilon}\} = 0$$

where $\varphi : \mathbb{P}^N \to \mathbb{P}^N$ is a morphism of degree ≥ 2 and if $O_\varphi(P)$ is Zariski dense in $\mathbb{P}^N$.

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MSC:

11G35	Varieties over global fields
11B37	Recurrences
14G40	Arithmetic varieties and schemes; Arakelov theory; heights
37F10	Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets

Keywords:
Arithmetic dynamical systems; orbit modulo p

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