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**Local bifurcations of critical periods for cubic Liénard equations with cubic damping.** (English) Zbl 1163.34349

J. Comput. Appl. Math. 222, No. 2, 404-410 (2008).

The paper is devoted to the local bifurcation of critical periods near a nondegenerate center $O(0,0)$ of the Liénard system

$$\dot{x} = y, \quad \dot{y} = -g(x)y - f(x)$$

with $f(x) = a_1x + a_2x^2 + a_3x^3$ and $g(x) = x + b_2x^2 + b_3x^3$, where $a_1, a_2, a_3, b_2, b_3 \in \mathbb{R}$. The authors first apply the results from *C. Christopher* and *J. Devlin* [J. Differ. Equations 200, No. 1, 1–17 (2004; Zbl 1059.34020)] to give a necessary and sufficient condition for the coefficients under which the cubic Liénard system with cubic damping has a center at $O$ and finding the set of coefficients in which the center is isochronous. It is proved that at most 2 local critical periods can be produced from either a weak center of finite order or the linear isochronous center and that at most one local critical period can be produced from nonlinear isochronous centers.

Reviewer: Alexander Grin (Grodno)

**MSC:**

34C23 Bifurcation theory for ordinary differential equations
34C05 Topological structure of integral curves, singular points, limit cycles of ordinary differential equations
34C25 Periodic solutions to ordinary differential equations



**Keywords:**

Liénard system; weak center; isochronous center; bifurcation

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