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Using dynamic programming with adaptive grid scheme for optimal control problems in economics. (English) Zbl 1202.49026

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Summary: The study of the solutions of dynamic models with optimizing agents has often been limited by a lack of available analytical techniques to explicitly find the global solution paths. On the other hand, the application of numerical techniques such as dynamic programming to find the solution in interesting regions of the state was restricted by the use of fixed grid size techniques. Following *L. Grüne* [Numer. Math. 75, No. 3, 319–337 (1997; Zbl 0880.65045); Numer. Math. 99, No. 1, 85–112 (2004; Zbl 1074.65009)], in this paper an adaptive grid scheme is used for finding the global solutions of discrete time Hamilton-Jacobi-Bellman equations. Local error estimates are established and an adapting iteration for the discretization of the state space is developed. The advantage of the use of adaptive grid scheme is demonstrated by computing the solutions of one- and two-dimensional economic models which exhibit steep curvature, complicated dynamics due to multiple equilibria, thresholds (Skiba sets) separating domains of attraction and periodic solutions. We consider deterministic and stochastic model variants. The studied examples are from economic growth, investment theory, environmental and resource economics.

MSC:

49L20	Dynamic programming in optimal control and differential games
49N90	Applications of optimal control and differential games
91B55	Economic dynamics
49M25	Discrete approximations in optimal control
90C39	Dynamic programming

Keywords:

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