

Ćirić, Ljubomir

Fixed point theorems for multi-valued contractions in complete metric spaces. (English)

[Zbl 1213.54063](#)

J. Math. Anal. Appl. 348, No. 1, 499-507 (2008).

Let (X, d) be a metric space and $\text{Cl}(X)$ the collection of nonempty closed sets of X . The main results of this paper are two fixed point theorems for multivalued functions. The following theorem is a generalization of Theorem 1 of *N. Mizoguchi* and *W. Takahashi* [*J. Math. Anal. Appl.* 141, No. 1, 177–188 (1989; [Zbl 0688.54028](#))].

Theorem A. Let (X, d) be a complete metric space and $T : X \rightarrow \text{Cl}(X)$ be a mapping of X into itself. If there exists a function $\varphi : [0, \infty) \rightarrow (0, 1)$ satisfying $\limsup \varphi * r < 1$ for each $t \in [0, \infty)$ and $r \rightarrow t+$ such that for any $x \in X$ there exists $y \in T(x)$ satisfying the following two conditions: $d(x, y) \leq (2 - \varphi(d(x, y)))D(x, Tx)$ and $D(y, T(y)) \leq \varphi(d(x, y))d(x, y)$, then T has a fixed point in X provided $f(x) = D(x, T(x))$ is lower-semicontinuous.

The following theorem is a generalization of Theorem 1 [op. cit.], of Theorem 2 of *Y. Feng* and *S. Liu* [*J. Math. Anal. Appl.* 317, No. 1, 103–112 (2006; [Zbl 1094.47049](#))] and *D. Klim* and *D. Wardowski* [*J. Math. Anal. Appl.* 334, No. 1, 132–139 (2007; [Zbl 1133.54025](#))].

Theorem B. Let (X, d) be a complete metric space and $T : X \rightarrow \text{Cl}(X)$ be a mapping of X into itself. If there exists a function $\varphi : [0, \infty) \rightarrow (0, 1)$ and a nondecreasing function $b : [0, \infty) \rightarrow [b, 1)$, $b > 0$, such that $\varphi(t) < b(t)$ and $\limsup_{t \rightarrow r+} \varphi(t) < \limsup_{t \rightarrow r+} b(t)$ for all $t \in [0, \infty)$, and for any $x \in X$ there exists $y \in T(x)$ satisfying the following conditions: $b(d(x, y))d(x, y) \leq D(x, Tx)$ and $D(y, T(y)) \leq \varphi(d(x, y))d(x, y)$, then T has a fixed point in X provided $f(x) = D(x, T(x))$ is lower-semicontinuous.

In the last part of the paper the author constructs two examples which show that the results from this paper are genuine generalizations of the results of Mizoguchi and Takahasi, Feng and Liu and Klim and Wardowski.

Reviewer: Valeriu Popa (Bacău)

MSC:

[54H25](#) Fixed-point and coincidence theorems (topological aspects)

[54C60](#) Set-valued maps in general topology

Cited in **2** Reviews
Cited in **38** Documents

Keywords:

complete metric space; proximal set; Hausdorff metric; set-valued contraction; fixed point

Full Text: [DOI](#)

References:

- [1] Ćirić, L.B.; Ume, J.S., Some common fixed point theorems for weakly compatible mappings, *J. math. anal. appl.*, 314, 2, 488-499, (2006) · [Zbl 1086.54027](#)
- [2] Ćirić, L.B., Common fixed point theorems for multi-valued mappings, *Demonstratio math.*, 39, 2, 419-428, (2006) · [Zbl 1112.47047](#)
- [3] Ćirić, L.B., Fixed point theorems for set-valued non-self mappings, *Math. balkanica*, 20, 2, 207-217, (2006) · [Zbl 1144.47325](#)
- [4] Daffer, P.Z.; Kaneko, H., Fixed points of generalized contractive multi-valued mappings, *J. math. anal. appl.*, 192, 655-666, (1995) · [Zbl 0835.54028](#)
- [5] Feng, Y.; Liu, S., Fixed point theorems for multi-valued contractive mappings and multi-valued Caristi type mappings, *J. math. anal. appl.*, 317, 103-112, (2006) · [Zbl 1094.47049](#)
- [6] Klim, D.; Wardowski, D., Fixed point theorems for set-valued contractions in complete metric spaces, *J. math. anal. appl.*, 334, 132-139, (2007) · [Zbl 1133.54025](#)
- [7] Mizoguchi, N.; Takahashi, W., Fixed point theorems for multivalued mappings on complete metric spaces, *J. math. anal. appl.*, 141, 177-188, (1989) · [Zbl 0688.54028](#)

- [8] Nadler, S.B., Multi-valued contraction mappings, Pacific J. math., 30, 475-488, (1969) · [Zbl 0187.45002](#)
- [9] Naidu, S.V.R., Fixed-point theorems for a broad class of multimaps, Nonlinear anal., 52, 961-969, (2003) · [Zbl 1029.54049](#)
- [10] Reich, S., Fixed points of contractive functions, Boll. unione mat. ital., 5, 26-42, (1972) · [Zbl 0249.54026](#)
- [11] Reich, S., Some fixed point problems, Atti acad. naz. lincei, 57, 194-198, (1974) · [Zbl 0329.47019](#)
- [12] Zhong, C.K.; Zhu, J.; Zhao, P.H., An extension of multi-valued contraction mappings and fixed points, Proc. amer. math. soc., 128, 2439-2444, (2000) · [Zbl 0948.47058](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.