Hmidi, Taoufik; Keraani, Sahbi

On the global well-posedness of the two-dimensional Boussinesq system with a zero diffusivity. (English) Zbl 1154.35073

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The authors consider the 2D Boussinesq system

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} - \nu \Delta \vec{v} + \nabla p = \theta \vec{e},$$

$$\partial_t \theta + \vec{v} \cdot \nabla \theta - \kappa \Delta \theta = 0,$$

$$\operatorname{div} \vec{v} = 0,$$

with initial conditions

$$(\vec{v}, \theta)|_{t=0} = \left(\vec{v}^0, \theta^0\right),$$

where $\vec{e} = (0,1)$, $\vec{v} = (v_1, v_2)$ is the velocity, p is the pressure, the kinematic viscosity ν is a positive parameter and the diffusivity coefficient κ is nonnegative. In this article, they study the specific situation where the temperature is only advected by the flow without diffusion (the molecular conductivity κ is zero). They first prove provided that $\theta^0 \in L^2$ and $\vec{v}^0$ is a divergence-free H^s-vector-field with $s \in [0,2)$, that system (BS) admits a global weak solution such that

$$\vec{v} \in C(\mathbb{R}_+; H^s) \cap L^2_{\text{loc}}(\mathbb{R}_+; H^{\min\{s+1,2\}}),$$

and

$$\theta \in C_b(\mathbb{R}_+; L^2).$$

Moreover, if $\theta^0 \in B^0_{2,1} \cap B^0_{p,\infty}$ with $p \in (2,\infty]$ and $\vec{v}^0$ is a divergence-free H^s-vector-field with $s \in (0,2]$, the system (BS) admits a unique global solution such that

$$\vec{v} \in C(\mathbb{R}_+; H^s) \cap L^2_{\text{loc}}(\mathbb{R}_+; H^{\min\{s+1,2\}}) \cap L^1_{\text{loc}}(\mathbb{R}_+; B^2_{2,1}),$$

and

$$\theta \in C(\mathbb{R}_+; B^0_{2,1} \cap B^0_{p,\infty}).$$

Reviewer: Bernard Ducomet (Bruyères le Châtel)

MSC:

35Q35 PDEs in connection with fluid mechanics
35B05 Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids

Keywords:

Boussinesq system; zero diffusivity; global weak solution