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Non-unique ergodicity, observers' topology and the dual algebraic lamination for $\mathbb{R}$-trees. (English) Zbl 1197.20020

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Summary: Let T be an $\mathbb{R}$-tree with a very small action of a free group F_N which has dense orbits. Such a tree T or its metric completion $\overline{T}$ are not locally compact. However, if one adds the Gromov boundary ∂T to $\overline{T}$, then there is a coarser 'observers' topology' on the union $\overline{T} \cup \partial T$, and it is shown here that this union, provided with the observers' topology, is a compact space $\widehat{T}^{\text{obs}}$.

To any $\mathbb{R}$-tree T as above a 'dual lamination' $L^2(T)$ has been associated by the authors [in J. Lond. Math. Soc., II. Ser. 78, No. 3, 737-754 (2008; Zbl 1198.20023)]. Here we prove that, if two such trees T_0 and T_1 have the same dual lamination $L^2(T_0) = L^2(T_1)$, then with respect to the observers' topology the two trees have homeomorphic compactifications: $\widehat{T}_0^{\text{obs}} = \widehat{T}_1^{\text{obs}}$. Furthermore, if both T_0 and T_1, say with metrics d_0 and d_1, respectively, are minimal, this homeomorphism restricts to an F_N-equivariant bijection $T_0 \to T_1$, so that on the identified set $T_0 = T_1$ one obtains a well defined family of metrics $\lambda d_1 + (1-\lambda) d_0$. We show that for all $\lambda \in [0,1]$ the resulting metric space T_λ is an $\mathbb{R}$-tree.

MSC:

20E08 Groups acting on trees
20E05 Free nonabelian groups
20F65 Geometric group theory
20F69 Asymptotic properties of groups
37A25 Ergodicity, mixing, rates of mixing
57M07 Topological methods in group theory
37E25 Dynamical systems involving maps of trees and graphs

Keywords:

$\mathbb{R}$-trees; actions of free groups; dense orbits; metric completions; Gromov boundaries; observer topologies; dual laminations

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