Bürger, Raimund; **García, Antonio**; **Karlsen, Kenneth H.**; **Towers, John D.**
Difference schemes, entropy solutions, and speedup impulse for an inhomogeneous kinematic traffic flow model. (English) Zbl 1173.35586
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Authors' abstract: The classical Lighthill-Whitham-Richards (LWR) kinematic traffic model is extended to a unidirectional road on which the maximum density $a(x)$ represents road inhomogeneities, such as variable numbers of lanes, and is allowed to vary discontinuously. The car density $\varphi(x,t)$ is then determined by the following initial value problem for a scalar conservation law with a spatially discontinuous flux:

$$\varphi_t + (\varphi v(\varphi/a(x))_x = 0, \quad \varphi(x,0) = \varphi_0(x), \qquad x \in \mathbb{R},\ t \in (0,T), \tag{*}$$

where $v(z)$ is the velocity function. We adapt to (*) a new notion of entropy solutions, which involves a Kružkov-type entropy inequality based on a specific flux connection (A,B), and which we interpret in terms of traffic flow. This concept is consistent with both the driver's ride impulse and the desire of drivers to speed up.

We prove that entropy solutions of type (A,B) are unique. This solution concept also leads to simple, transparent, and unified convergence proofs for numerical schemes. Indeed, we adjust to (*) new variants of the Engquist- Osher (EO) scheme and of the Hilliges-Weidlich (HW) scheme analyzed by the authors. It is proven that the EO and HW schemes and a related Godunov scheme converge to the unique entropy solution of type (A,B) of (*). For the Godunov version, this is the first rigorous convergence and well-posedness result, since no unnecessarily restrictive regularity assumptions are imposed on the solution. Numerical experiments for first-order schemes and formally second-order MUSCL/Runge-Kutta versions are presented.

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MSC:

35L65	Hyperbolic conservation laws
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
90B20	Traffic problems in operations research
35L45	Initial value problems for first-order hyperbolic systems

Keywords:

discontinuous flux; entropy condition; driver's ride impulse; Lighthill-Whitham-Richards (LWR); first-order schemes; second-order MUSCL/Runge-Kutta versions

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