

**Towers, John D.**

**A convergence rate theorem for finite difference approximations to delta functions.** (English)

Zbl 1155.65016

J. Comput. Phys. 227, No. 13, 6591-6597 (2008).

A new rate of convergence for approximations to certain integrals over codimension one manifolds in  $\mathbb{R}^n$ , is proved. The type of manifold is defined by the zero level set of a smooth mapping  $u : \mathbb{R}^n \rightarrow \mathbb{R}$ . The approximation method used, is based on two finite difference methods for the discretizing of the delta function, originally presented by the author in a previous research work [J. Comput. Phys. 220, No. 2, 915–931 (2007; Zbl 1115.65028)], where empirical convergence rates had indicated the first and second order accuracy. In this work these convergence rates are proved for the two proposed algorithms, under fairly general hypotheses.

Reviewer: [Vasilis Dimitriou \(Chania\)](#)

**MSC:**

**65D15** Algorithms for approximation of functions

Cited in **6** Documents

**Keywords:**

delta function; integral; level set method; finite difference; approximation; regular grid; convergence rate

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**References:**

- [1] J.T. Beale, A proof that a discrete delta function is second order accurate, Preprint at <<http://www.math.duke.edu/beale/papers/ddel.pdf>>. · Zbl 1136.65017
- [2] V.F. Candela, A. Marquina, On the numerical approximation of the length of (implicit) level curves, Preprint 2006. · Zbl 1203.65040
- [3] Chang, Y.C.; Hou, T.Y.; Merriman, B.; Osher, S., A level set formulation of Eulerian interface capturing methods for incompressible fluid flows, J. comput. phys., 124, 449-464, (1996) · Zbl 0847.76048
- [4] Engquist, B.; Tornberg, A.K.; Tsai, R., Discretization of Dirac delta functions in level set methods, J. comput. phys., 207, 28-51, (2005) · Zbl 1074.65025
- [5] Liu, H.; Osher, S.; Tsai, R., Multi-valued solution and level set methods in computational high frequency wave propagation, Commun. comput. phys., 1, 5, 765-804, (2006) · Zbl 1120.65110
- [6] Mayo, A., The fast solution of poisson's and the biharmonic equations on irregular regions, SIAM J. numer. anal., 21, 285-299, (1984) · Zbl 1131.65303
- [7] Min, C.; Gibou, F., Geometric integration over irregular domains with application to level-set methods, J. comput. phys., 226, 1432-1443, (2007) · Zbl 1125.65021
- [8] Osher, S.; Fedkiw, R., Level set methods and dynamic implicit surfaces, (2003), Springer-Verlag New York · Zbl 1026.76001
- [9] Osher, S.; Sethian, J., Fronts propagating with curvature dependent speed: algorithms based on hamilton – jacobi formulations, J. comput. phys., 79, 12-49, (1988) · Zbl 0659.65132
- [10] Peng, D.; Merriman, B.; Osher, S.; Zhao, H.; Kang, M., A PDE-based fast local level set method, J. comput. phys., 155, 410-438, (1999) · Zbl 0964.76069
- [11] Rumborg, O., Mathematical models and numerical methods for high frequency waves, Commun. comput. phys., 2, 5, 827-880, (2007) · Zbl 1164.78300
- [12] Sethian, J.A., Level set methods and fast marching methods, (1999), Cambridge University Press Cambridge · Zbl 0929.65066
- [13] Smereka, P., The numerical approximation of a delta function with application to level set methods, J. comput. phys., 211, 77-90, (2006) · Zbl 1086.65503
- [14] Tornberg, A.K.; Engquist, B., Numerical approximations of singular source terms in differential equations, J. comput. phys., 200, 462-488, (2004) · Zbl 1115.76392
- [15] Tornberg, A.K.; Engquist, B., Regularization techniques for numerical approximation of PDEs with singularities, J. sci. comput., 19, 527-552, (2003) · Zbl 1035.65085
- [16] Towers, J.D., Two methods for discretizing a delta function supported on a level set, J. comput. phys., 220, 915-931, (2007) ·

[Zbl 1115.65028](#)

- [17] J.D. Towers, Discretizing delta functions via finite difference and gradient normalization, Preprint at <http://www.miracosta.edu/home/jtowers/>.  
· [Zbl 1167.65007](#)
- [18] Wen, X., High order numerical methods to a type of delta function integrals, J. comput. phys., 226, 1952-1967, (2007) · [Zbl 1125.65024](#)
- [19] X. Wen, High order numerical quadratures to one dimensional delta function integrals, Preprint at <http://lsec.cc.ac.cn/wenxin/research.html>.  
· [Zbl 1170.65008](#)

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