zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Isotalo, Jarkko; **Puntanen, Simo**; **Styan, George P. H.**
The BLUE's covariance matrix revisited: A review. (English) Zbl 1141.62325
J. Stat. Plann. Inference 138, No. 9, 2722-2737 (2008).

Summary: We comment on and review some unexpected but interesting features of the BLUE (best linear unbiased estimator) of the expectation vector in the general linear model and in particular the BLUE's covariance matrix. Most of these features appear in the literature but are rather scattered or hidden.

MSC:

62H12 Estimation in multivariate analysis
62J05 Linear regression; mixed models
62H20 Measures of association (correlation, canonical correlation, etc.)

Keywords:

arithmetic mean; best linear unbiased estimator (BLUE); canonical correlations; generalized inverse; linear model; minimum variance linear unbiased estimator; ordinary least squares estimator (OLSE); orthogonal projector; proper eigenvalues; residuals; Watson efficiency

Full Text: DOI

References:

[1] Anderson, T.W., On the theory of testing serial correlation, Skandinavisk aktuarietidskrift, 31, 88-116, (1948) · Zbl 0033.08001
[2] Anderson, T.W., An introduction to multivariate statistical analysis, (2003), Wiley New York · Zbl 0083.14601
[3] Anderson, W.N., Shorted operators, SIAM J. appl. math., 20, 520-525, (1971) · Zbl 0217.05503
[4] Anderson, W.N.; Trapp, G.E., Shorted operators, II, SIAM J. appl. math., 28, 60-71, (1975) · Zbl 0295.47032
[5] Baksalary, J.K.; Puntanen, S., Spectrum and trace invariance criterion and its statistical applications, Linear algebra appl., 142, 121-128, (1990) · Zbl 0714.15002
[6] Baksalary, J.K., Puntanen, S., Styan, G.P.H., 1990a. On T.W. Anderson's contributions to solving the problem of when the ordinary least-squares estimator is best linear unbiased and to characterizing rank additivity of matrices. In: Styan, G.P.H. (Ed.), The Collected Papers of T.W. Anderson: 1943-1985. Wiley, New York, pp. 1579-1591.
[7] Baksalary, J.K.; Puntanen, S.; Styan, G.P.H., A property of the dispersion matrix of the best linear unbiased estimator in the general gauss – markov model, Sankhyā, ser. A, 52, 279-296, (1990) · Zbl 0727.62072
[8] Baksalary, J.K.; Puntanen, S.; Yanai, H., Canonical correlations associated with symmetric reflexive generalized inverses of the dispersion matrix, Linear algebra appl., 176, 61-74, (1992) · Zbl 0766.62032
[9] Bartmann, F.C.; Bloomfield, P., Inefficiency and correlation, Biometrika, 68, 67-71, (1981) · Zbl 0472.62073
[10] Blom, G., When is the arithmetic Mean blue?, The American Statistician, 30, 40-42, (1976) · Zbl 0324.62027
[11] Chu, K.L.; Isotalo, J.; Puntanen, S.; Styan, G.P.H., On decomposing the Watson efficiency of ordinary least squares in a partitioned weakly singular linear model, Sankhyā, 66, 634-651, (2004) · Zbl 1193.62094
[12] Chu, K.L.; Isotalo, J.; Puntanen, S.; Styan, G.P.H., Some further results concerning the decomposition of the Watson efficiency in partitioned linear models, Sankhyā, 67, 74-89, (2005) · Zbl 1193.62095
[13] DeGroot, M.H., A conversation with T.W. Anderson, Statist. sci., 1, 97-105, (1986) · Zbl 0955.01511
[14] de Leeuw, J., Generalized eigenvalue problems with positive semi-definite matrices, Psychometrika, 47, 87-93, (1982) · Zbl 0487.65020
[15] Drury, S.W.; Liu, S.; Lu, C.-Y.; Puntanen, S.; Styan, G.P.H., Some comments on several matrix inequalities with applications to canonical correlations: historical background and recent developments, Sankhyā, ser. A, 64, 453-507, (2002) · Zbl 1192.15007
[16] Golub, G.H.; Van Loan, C.F., Matrix computations, (1996), Johns Hopkins University Press Baltimore, MD · Zbl 0865.65009
[17] Khatri, C.G., A note on multiple and canonical correlation for a singular covariance matrix, Psychometrika, 41, 465-470, (1976) · Zbl 0381.62048
[18] Khatri, C.G., Some properties of BLUE in a linear model and canonical correlations associated with linear transformations, J. multivariate anal., 34, 211-226, (1990) · Zbl 0731.62116
[19] Kreı˘n, M.G., The theory of self-adjoint extensions of semi-bounded Hermitian transformations and its applications, I, II (in Russian), Matematiceskiı˘ sbornik novaya seriya˘, 20, 62, 431-495, (1947), \& 21(63), 365-404 · Zbl 0029.14103

[20] Latour, D., Puntanen, S., Styan, G.P.H., 1987. Equalities and inequalities for the canonical correlations associated with some partitioned generalized inverses of a covariance matrix. In: Pukkila T., Puntanen, S. (Eds.), Proceedings of the Second International Tampere Conference in Statistics. University of Tampere, Tampere, Finland, 1-4 June 1987, vol. A184, Department of Mathematical Sciences/Statistics, University of Tampere, Tampere, Finland, pp. 541-553.

[21] Liski, E.P.; Puntanen, S., A further note on a theorem on the difference of the generalized inverses of two nonnegative definite matrices, Commun. statist. theory and methods, 18, 1747-1751, (1989) · Zbl 0696.62268

[22] Marsaglia, G.; Styan, G.P.H., Equalities and inequalities for ranks of matrices, Linear and multilinear algebra, 2, 269-292, (1974)

[23] Marshall, A.W.; Olkin, I., Inequalities: theory of majorization and its applications, (1979), Academic Press New York · Zbl 0437.26007

[24] McDonald, R.P.; Torii, Y.; Nishisato, S., Some results on proper eigenvalues and eigenvectors with applications to scaling, Psychometrika, 44, 211-227, (1979) · Zbl 0423.62088

[25] Mitra, S.K.; Moore, B.J., Gauss – markov estimation with an incorrect dispersion matrix, Sankhyā, ser. A, 35, 139-152, (1973) · Zbl 0277.62044

[26] Mitra, S.K.; Puntanen, S., The shorted operator statistically interpreted, Calcutta statist. assoc. bull., 40, 97-102, (1991) · Zbl 0744.62080

[27] Mitra, S.K.; Puri, M.L., Shorted operators and generalized inverses of matrices, Linear algebra appl., 25, 45-56, (1979) · Zbl 0447.15002

[28] Mitra, S.K.; Rao, C.R., Simultaneous reduction of a pair of quadratic forms, Sankhyā ser. A, 30, 313-322, (1968) · Zbl 0198.35204

[29] Mitra, S.K., Puntanen, S., Styan, G.P.H., 1995. Shorted matrices and their applications in linear statistical models: a review. In: Tiit, E.-M., Kollo, T., Niemi, H., (Eds.), Multivariate Statistics and Matrices in Statistics: Proceedings of the 5th Tartu Conference, Tartu-Pühajärve, Estonia, 23-28 May 1994, New Trends in Probability and Statistics, vol. 3. VSP, Utrecht, Netherlands \& TEV, Vilnius, Lithuania, pp. 289-311.

[30] Puntanen, S., 1986. Properties of the covariance matrix of the BLUE in the general linear model. In: Francis, I.S., Manly, B.F.J., Lam, F.C., (Eds.), Pacific Statistical Congress: Auckland, New Zealand, May 1985. Elsevier, Amsterdam, pp. 425-430.

[31] Puntanen, S., 1987. On the relative goodness of ordinary least squares estimation in the general linear model. Ph.D. Dissertation, Department of Mathematical Sciences, University of Tampere, Tampere, Finland, Acta Universitatis Tamperensis, Ser. A, 216. · Zbl 0652.62050

[32] Puntanen, S.; Styan, G.P.H., The equality of the ordinary least squares estimator and the best linear unbiased estimator [with comments by oscar kempthorne \& by shayle R. searle and with "reply" by the authors], The American Statistician, 43, 153-164, (1989)

[33] Rao, C.R., 1967. Least squares theory using an estimated dispersion matrix and its application to measurement of signals. In: Le Cam, L.M., Neyman, J. (Eds.), Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability: Berkeley, California, 1965/1966, vol. 1. University of California Press, Berkeley, pp. 355-372.

[34] Rao, C.R., Unified theory of linear estimation, Sankhyā, ser. A, 33, 371-394, (1971), [Corrigendum (1972), 34, pp. 194, 477.] · Zbl 0236.62048

[35] Rao, C.R.; Mitra, S.K., Generalized inverse of matrices and its applications, (1971), Wiley New York

[36] Scott, A.J.; Styan, G.P.H., On a separation theorem for generalized eigenvalues and a problem in the analysis of sample surveys, Linear algebra appl., 70, 209-224, (1985) · Zbl 0587.62023

[37] Seshadri, V., Styan, G.P.H., 1980. Canonical correlations, rank additivity and characterizations of multivariate normality. In: Analytic Function Methods in Probability Theory: Proceedings of the Colloquium on the Methods of Complex Analysis in the Theory of Probability and Statistics held at the Kossuth L. University, Debrecen, Hungary, August 29-September 2, 1977. Colloquia Mathematica Societatis János Bolyai, vol. 21, János Bolyai, Budapest and North-Holland, Amsterdam, pp. 331-344.

[38] Stewart, G.W.; Sun, J.-G., Matrix perturbation theory, (1990), Academic Press San Diego, CA

[39] Styan, G.P.H., 1985. Schur complements and linear statistical models. In: Pukkila, T., Puntanen, S. (Eds.), Proceedings of the First International Tampere Seminar on Linear Statistical Models and their Applications: Tampere, Finland, August-September 1983. Department of Mathematical Sciences/Statistics, University of Tampere, Tampere, Finland, pp. 37-75.

[40] Watkins, D.S., 2007. Chapter 43: Unsymmetric matrix eigenvalue techniques. In: Hogben, L. (Ed.), Handbook of Linear Algebra, Chapman \& Hall/CRC, Boca Raton, FL.

[41] Watson, G.S., Serial correlation in regression analysis I, Biometrika, 42, 327-341, (1955) · Zbl 0068.33201

[42] Zyskind, G., On canonical forms, non-negative covariance matrices and best and simple least squares linear estimators in linear models, Ann. math. statist., 38, 1092-1109, (1967) · Zbl 0171.17103

[43] Zyskind, G., Parametric augmentations and error structures under which certain simple least squares and analysis of variance procedures are also best, J. amer. statist. assoc., 64, 1353-1368, (1969)

