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Gaps in the exponent spectrum of subgroups of discrete quasiconformal groups. (English)

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The authors study a class of convergence groups, discrete quasiconformal groups, acting on the n -dimensional unit ball \mathbb{B}^n . A group G of quasiconformal homeomorphisms is a discrete quasiconformal group if there exists a uniform bound for the dilatation coefficients of all elements of G . For example, a Kleinian group acting on \mathbb{B}^n can be regarded as a discrete 1-quasiconformal group. The main result is the following. Denote by $\Lambda_c^s(G)$ the set of strong conical limit points of G , i.e. points $\zeta \in S^{n-1}$ for which there exists a sequence (g_j) in G such that $(g_j(0))$ converges to ζ within a Euclidean cone based at ζ , and $(g_j^{-1}(0))$ converges to a point $b \neq \zeta$. The set of all K -fat horospherical limit points of G , $\Lambda_K(G)$, is the set of all points $\zeta \in S^{n-1}$ for which there exists a sequence (g_j) in G and a constant $C > 0$ such that

$$\frac{1 - |g_j(0)|}{|g_j(0) - \zeta|^{K+1}} \geq C$$

for all j . Suppose that G is a discrete K -quasiconformal group acting on \mathbb{B}^n with empty regular set, and let \widehat{G} be a non-elementary normal subgroup of G . Then

$$\Lambda_c^s(G) \subset \Lambda_K(\widehat{G}).$$

For $n = 3$, the authors show that for a discrete quasiconformal group G acting on \mathbb{B}^3 , with empty regular set and purely conical limit set, the strong conical limit set of G has full 2-dimensional Lebesgue measure in S^2 . As an application, a lower bound for the exponent of convergence of a non-elementary normal subgroup \widehat{G} of G is given.

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MSC:

30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)

30C65 Quasiconformal mappings in \mathbb{R}^n , other generalizations

Keywords:

conical limit set; exponent of convergence; Hausdorff dimension

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