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Asymptotic properties of bridge estimators in sparse high-dimensional regression models.
(English) [Zbl 1133.62048](#)
Ann. Stat. 36, No. 2, 587-613 (2008).

Summary: We study the asymptotic properties of bridge estimators in sparse, high-dimensional, linear regression models when the number of covariates may increase to infinity with the sample size. We are particularly interested in the use of bridge estimators to distinguish between covariates whose coefficients are zero and covariates whose coefficients are nonzero. We show that under appropriate conditions, bridge estimators correctly select covariates with nonzero coefficients with probability converging to one and that the estimators of nonzero coefficients have the same asymptotic distribution that they would have if the zero coefficients were known in advance. Thus, bridge estimators have an oracle property in the sense of *J. Fan and R. Li* [*J. Am. Stat. Assoc.* 96, No. 456, 1348–1360 (2001; [Zbl 1073.62547](#))] and *J. Fan and H. Peng* [*Ann. Stat.* 32, No. 3, 928–961 (2004; [Zbl 1092.62031](#))]. In general, the oracle property holds only if the number of covariates is smaller than the sample size. However, under a partial orthogonality condition in which the covariates of the zero coefficients are uncorrelated or weakly correlated with the covariates of nonzero coefficients, we show that marginal bridge estimators can correctly distinguish between covariates with nonzero and zero coefficients with probability converging to one even when the number of covariates is greater than the sample size.

MSC:

[62J05](#) Linear regression; mixed models
[62F12](#) Asymptotic properties of parametric estimators
[62E20](#) Asymptotic distribution theory in statistics
[62J07](#) Ridge regression; shrinkage estimators (Lasso)
[62H12](#) Estimation in multivariate analysis

Cited in **1** Review
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Keywords:

penalized regression; high-dimensional data; variable selection; asymptotic normality; oracle property

Full Text: [DOI](#) [arXiv](#)

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