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An introduction to involutive structures. (English) Zbl 1151.35011

New Mathematical Monographs 6. Cambridge: Cambridge University Press (ISBN 978-0-521-87857-9/hbk). xii, 392 p. (2008).

This book introduces the reader to a number of results on systems of vector fields with complex-valued coefficients defined on a smooth manifold M. It is assumed that the involutive structure (M, V) is locally integrable, where V is an involutive sub-bundle of the complexified tangent bundle CTM. It is assumed that the involutive structure (M, V) is locally integrable, that is, the orthogonal of V, which is a sub-bundle of the complexified cotangent bundle CT^*M, is locally generated by exact differentials. When (M, V) locally integrable, each point has a neighborhood U such that if $\{L_1, \ldots, L_m\}$ are n smooth vector fields that form a basis of V over U, then we can find $m = \dim M - n$ smooth, complex-valued functions $Z_1, \ldots, Z_m$ such that are solutions of the equation

$$L_j h = 0, \quad 1 \leq j \leq n \tag{1}$$

and whose differentials are linearly independent over C at each point U.

Chapter I introduces the basic concepts in the theory involutive and locally integrable structures, CR structure, elliptic structure, and real analytic structure are identified and examples are provided. Chapter II devoted to the approximation theorem of Baouendi and Treves which means that in a locally integrable structure, each solution of (1) can be locally approximated by a sequence $P_K(Z)$, where the P_K are holomorphic polynomials of m variables and Z is a complete set of first integrals. It is shown that the approximation is valid in many function spaces in analysis. Chapter III presents a variety of results on unique continuation for solutions and approximate solutions in a locally integrable structure (M, V). Chapter IV provides a detailed treatment of locally solvable vector fields. In the chapter, where the focus is on planar vector fields, the solvability condition (P) of Nirenberg and Treves is discussed and a priori estimates are proved in L^p and in a mixed norm that involudes the Hardy space $h^1(\mathbb{R})$. The first part of Chapter V introduces certain submanifolds in an involutive structure (M, V) which are important in the study of solutions, which are generalizations of the totally real and generic CR submanifolds. The second part of the chapter introduces the FBI transform first in $\mathbb{R}^n$ and then in a locally integral structure. The FBI transform is then applied to derive edge-of-the wedge type results. Chapter VI studies some boundary properties of the solutions of local integrable vector fields. These properties include the existence of a trace at the boundary, pointwise convergence of solutions to their boundary value, and the validity of Hardy space-like properties. Chapter VII describes the differential complex attached to a general involutive structure. Chapter VIII deals with the local solvability theory of the undetermined systems of partial differential equations naturally associated with a locally integrable structure, that is, the cohomology theory of its differential complex.

Finally, the authors conclude with an epilogue which summarizes some of the results obtained in recent years on diverse areas such as the similar principle, Mizohata structure, and hyperfunction solutions in hypoanalytic manifolds.

Reviewer: K. Kajitani (Ibaraki)

MSC:

35F05 Linear first-order PDEs

Keywords:

involutive structure; locally integrable approximation theory of Baouendi-Treves local solvability; FBI transform