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Constructible sets of linear differential equations and effective rational approximations of polylogarithmic functions. (English) [Zbl 1143.34057](#)

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The paper is concerned with rational approximations to solutions of certain Fuchsian linear differential equations defined on the Riemann sphere from the viewpoint of the theory of moduli of differential equations with a fixed monodromy group. The author finds a connection between Fuchsian ODE's and Diophantine approximations of special Siegel G -functions. In particular, the rigidity assumption for the Fuchsian ODE's enables the author to give new explicit formulae for Padé approximations.

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MSC:

- [34M35](#) Singularities, monodromy and local behavior of solutions to ordinary differential equations in the complex domain, normal forms Cited in 6 Documents
- [30E10](#) Approximation in the complex plane
- [33B30](#) Higher logarithm functions

Keywords:

Fuchsian linear differential equations

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