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Nonlinear equations involving nonpositive definite linear operators via variational methods. (English) Zbl 1147.47049

J. Integral Equations Appl. 19, No. 1, 1-12 (2007).

The paper is concerned with the application of variational methods, namely the variational method due to *B. Ricceri* [J. Comput. Appl. Math. 113, No. 1–2, 401–410 (2000; Zbl 0946.49001)] in showing the existence of solutions for nonlinear equations of the type $u = K\mathbf{f}(u)$, where $K : L^{q_0}(\Omega) \to L^{p_0}(\Omega)$ is a completely continuous linear operator, $\mathbf{f}(u) = f(\cdot, u(\cdot))$ denotes the superposition operator associated with f such that $\mathbf{f}(u) \in L^q(\Omega)$ for every $u \in L^p(\Omega)$ and $2 < p < p_0$, $\frac{1}{p} + \frac{1}{q} = 1$, $\frac{1}{p_0} + \frac{1}{q_0} = 1$.

The problems considered in this research were usually tackled by fixed point methods. The authors of the present study apply variational methods which improve the few known results obtained by critical point theory and which apply the action functional associated with the considered problem fails to be coercive. The authors, contrary to some other results in the field, do not assume K to be positive definite but require that it has a finite number of negative eigenvalues and put some weak assumption on the nonlinear term which provides that the action functional is not coercive. The results of the paper are applied to solving a Hammerstein integral equation and a nonresonant nonlinear Fredholm integral equation.

Reviewer: Marek Galewski (Łódź)

MSC:

47J05 Equations involving nonlinear operators (general)
49J40 Variational inequalities
47J30 Variational methods involving nonlinear operators
47H30 Particular nonlinear operators (superposition, Hammerstein, Nemytskiĭ, Uryson, etc.)
49J45 Methods involving semicontinuity and convergence; relaxation
49J50 Fréchet and Gateaux differentiability in optimization

Keywords:

variational method; nonlinear integral equation; Fredholm equation; Hammerstein equation

Full Text: DOI

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