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From N parameter fractional Brownian motions to N parameter multifractional Brownian motions. (English) Zbl 1135.60020

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Summary: Multifractional Brownian motion is an extension of the well-known fractional Brownian motion where the Hölder regularity is allowed to vary along the paths. In this paper, two kinds of multi-parameter extensions of mBm are studied: one is isotropic while the other is not. For each of these processes, a moving average representation, a harmonizable representation, and the covariance structure are given.

The Hölder regularity is then studied. In particular, the case of an irregular exponent function H is investigated. In this situation, the almost sure pointwise and local Holder exponents of the multi-parameter mBm are proved to be equal to the correspondent exponents of H. Eventually, a local asymptotic self-similarity property is proved. The limit process can be another process than fBm.

MSC:

60G15 Gaussian processes
60G05 Foundations of stochastic processes

Keywords:

Hölder regularity; local asymptotic self-similarity; multi-parameter processes

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.