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Introduction to group theory. Translated from the Russian. With a new chapter. (English)
Zbl 1215.20001
EMS Textbooks in Mathematics. Zürich: European Mathematical Society (ISBN 978-3-03719-041-8/hbk). x, 177 p. (2008).

From the back cover: "This book quickly introduces beginners to general group theory and then focuses on three main themes: finite group theory, including sporadic groups; combinatorial and geometric group theory, including the Bass-Serre theory of groups acting on trees; the theory of train tracks by Bestvina and Handel for automorphisms of free groups .... Presupposing only a basic knowledge of algebra, the book is addressed to anyone interested in group theory: from advanced undergraduate and graduate students to specialists." From the Preface: "The purpose of the book is to present the fundamentals of group theory and to describe some nontrivial constructions and techniques, which will be useful to specialists."

The text should appeal to group theorists who want to broaden their familiarity with the application found in the areas indicated above. A student with the background of a first course in graduate algebra together with an advanced linear algebra course could focus on the constructions and techniques for insight into the classification of finite simple groups, the Bass-Serre theory of groups acting on trees, and the theory of train tracks. In either of these a previous experience with trees and train tracks would facilitate the study. The group theoretic concepts are restricted to those necessary for immediate application.

Through a carefully selected choice of group theoretic concepts, the author has skillfully created access to areas of application that are nonstandard in a first course in group theory.

For example in Chapter 1, the rudiments associated with finite permutation groups are sufficient for the development of the fundamental results associated with the simplicity of the alternating group A_n, $n > 4$, including that A_5 is the nonabelian simple group of least order. Squeezed in between is consideration of A_5 as the rotation group of an icosahedron and included is A_5 as a projective special linear group which motivates the search for which n is $\mathrm{PSL}_n(K)$ over a field K simple. This is followed by the introduction of a projective plane in order to introduce Mathieu's group M_{22} which initiates a section on the examination of the interrelationship of Mathieu groups, Steiner systems, and coding theory. A brief section in extension theory permits proving Schur's theorem that each extension of a finite group H by a finite group F having relatively prime orders is splitting. Chapter 1 ends with proving the simplicity of the Higman-Sims group. This last development is dependent upon the earlier section on M_{22}. So within a chapter, a section can not always be arbitrarily selected. However the author indicates the second chapter (page 45) may be read independently of the first.

Chapter 2 is an introduction to combinatorial group theory based on the author's premise of studying the action of the group on an appropriate geometric object. In this regard the author begins with Cayley's graphs and the automorphisms of trees which provides a foundation upon which to develop free groups and their association with trees. This includes the fundamental group of the graph, the presentation of groups by generators and relations, and Tietze transformations followed by free groups together with an association of trees and amalgamated free products. The action of $\mathrm{SL}_2(\mathbb{Z})$ on the hyperbolic plane is presented. The HNN extensions follow together with the association of these extensions with trees and amalgamated products. Kurosh's theorem on free products amalgamated over a common subgroup is introduced. Coverings of graphs are examined followed by the standard results related to subgroups in free and finitely generated groups. The introduction of complexes leads to investigating Hopfian groups and residually finite groups which include the result that the word problem in any finitely presented residually finite group is solvable and the number of subgroups of finite index in a finitely generated group is finite. As a consequence, the chapter gives a broad and in-depth examination of free groups, free products, and those groups which are finitely presented. The development is interwined with complexes, their coverings, and surfaces.

These two chapters are the translation of the original version.

This English edition is extended by a third chapter: "Automorphisms of free groups and train tracks." It begins with Nielsen's method and generators of the free group F_n with n generators followed by maps of graphs, homotopy equivalence, topological representations, the transition matrix, train tracks, transformation of maps, the metric induced on a graph by an irreducible map, proof of the Bestvina-Handel theorem that every irreducible outer automorphism of F_n can be topologically represented by an irreducible train map, examples of the construction of train tracks, and two applications of train tracks. The Appendix presents the Perron-Frobenius theorem on nonnegative, nonzero, irreducible real matrices.

Examples are plentiful and well-chosen.

At times the exposition is tight. For example: "...(see Lemma 16.13, Exercise 16.14 and Proposition 6.8, Lemma 20.8),...."

Exercises are an integral and essential part of the theoretical development. Solutions to a selected set of exercises are provided in detail. Proving that M_{22} is not isomorphic to either A_n or $\mathrm{PSL}_n(q)$ for any n and q is not among them. By design, the text engages the reader as a willing and active participant in the conceptual development. For this reason, a priori experience with the concepts introduced, especially those of a geometric context, will smooth the transition toward achieving the author's goal. These comments are observations, not criticisms. The text should be welcomed by those who want exposure to the application of group theory within a nonstandard geometric environment.

Reviewer: Homer F. Bechtell (Durham)

MSC:

20-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to group theory

20Dxx Abstract finite groups
20F05 Generators, relations, and presentations of groups
20E05 Free nonabelian groups
20E36 Automorphisms of infinite groups
20F65 Geometric group theory
20E08 Groups acting on trees

Keywords:

finite group theory; combinatorial group theory; free groups; train tracks; geometric group theory; groups acting on trees; finitely generated groups

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