Broszka, Dorota; Grande, Zbigniew

**On $\mathcal{I}$-differentiation.** (English) Zbl 1164.26007

Tatra Mt. Math. Publ. 35, 25-40 (2007).

Let $\mathcal{K}$ be a field of all reals (or of all complex numbers) and let $\mathcal{I} \subset \mathcal{K}$ be a set such that 0 is an accumulation point of $\mathcal{I}$. Let $Y$ be a real (or a complex) Banach space. If $f\,\mathcal{K} \to Y$ is a function then $D_h^{\mathcal{I}} f(x_0) = \lim\limits_{\mathcal{I} \ni r \to 0} \frac{f(x_0+rh)-f(x_0)}{r}$ is called the $\mathcal{I}$-derivative of $f$ at the point $x_0 \in \mathcal{K}$ in the direction $h \in \mathcal{K}$ when ever this limit exists and belongs to $Y$. If $\mathcal{K} = \mathcal{I}$ and $h = 1$ then we obtain the ordinary derivative of $f$ at the point $x_0$. The limit $Ds_h^{\mathcal{I}} f(x_0) = \lim\limits_{\mathcal{K} \times \mathcal{I} \ni (x,r) \to (x_0,0)} \frac{f(x+rh)-f(x)}{r}$ is called strong $\mathcal{I}$-derivative of $f$ at $x_0 \in \mathcal{K}$ in the direction $h \in \mathcal{K}$. Some properties of $\mathcal{I}$- derivatives and strong $\mathcal{I}$-derivatives are investigated.

Reviewer: Ján Borsík (Košice)

**MSC:**

26A24 Differentiation (real functions of one variable): general theory, generalized derivatives, mean value theorems

**Keywords:**

$\mathcal{I}$-derivative; direction; strongly $\mathcal{I}$-derivative