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A home-made Hartshorne-Serre correspondence. (English) Zbl 1133.14046
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R. Hartshorne [Bull. Am. Math. Soc. 80, 1017–1032 (1974; [Zbl 0304.14005](#))] proved that a codimension two subvariety of \mathbb{P}^n is the zero locus of a rank two vector bundle if and only if the variety is subcanonical (i.e. the determinant of the normal bundle extends to a line bundle on \mathbb{P}^n). Beside Hartshorne’s proof, which is based on ideas of Serre and Horrocks, there is an independent proof by *W. Barth* and *A. Van de Ven* [Invent. Math. 25, 91–106 (1974; [Zbl 0295.14006](#))], while *H. Grauert* and *G. Müllich* [Manuscr. Math. 16, 75–100 (1975; [Zbl 0318.32027](#))] gave the same result on a more general ambient space [see also *M. Valenzano*, Rend. Semin. Mat. Univ. Pol. Torino 62, No. 3, 235–254 (2004; [Zbl 1183.14026](#))]. A more general result is due to Vogelaar who, in his PHD thesis, proved that any local complete intersection subscheme of codimension two of a smooth variety X is the dependency locus of $r - 1$ sections of a rank r vector bundle on X of determinant L if and only if the determinant of its normal bundle twisted with L^* is generated by $r - 1$ global sections (provided that a cohomological condition on L^* is fulfilled). The present paper investigates the general Hartshorne-Serre correspondence as it is considered in Vogelaar’s thesis (unpublished) and in Grauert’s and Müllich’s paper. The approach is very elementary and concrete: the vector bundle of Vogelaar’s theorem is built starting with the local representation of its sections and through a careful investigation of the transition functions and matrices. The paper has the aim of giving a reference for the Hartshorne-Serre correspondence, but has the additional merit of being clear and accessible also to mathematicians who are not expert of algebraic geometry.

Reviewer: [Paolo Valabrega \(Torino\)](#)

MSC:

[14M07](#) Low codimension problems in algebraic geometry
[14F05](#) Sheaves, derived categories of sheaves, etc. (MSC2010)

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