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On the growth of solutions of a class of higher order linear differential equations with coefficients having the same order. (English) Zbl 1141.34054

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The authors study the growth of solutions of the linear differential equation

$$f^{(k)} + A_{k-1}(z)f^{(k-1)}(z) + \cdots + A_0 f = 0, \tag{1}$$

where $A_0(z), \dots, A_{k-1}(z)$ are entire functions with $A_0(z) \not\equiv 0$ $(k \geq 2)$. It is well-known that all solution of (1) are entire functions. Let $\sigma(f)$ denote the order of growth of entire function $f(z)$, $\tau(f)$ to denote the type of $f(z)$ with $\sigma(f) = \sigma$ and use the notation $\sigma_2(f)$ to denote the hyper-order of $f(z)$.

Z.-X. Chen obtained the following result.

Proposition A. Let $A_j(z)$ $(j = 0, \dots, k-1)$ be entire functions such that

$$\max\{\sigma(A_j), j = 1, \dots, k-1\} < \sigma(A_0) < +\infty.$$

Then every solution $f \not\equiv 0$ of (1) satisfies $\sigma_2(f) = \sigma(A_0)$.

The authors proved the following theorem

Theorem 1. Let $A_j(z)$ $(j = 0, \dots, k-1)$ be entire functions satisfying $\sigma(A_0) = \sigma$, $\tau(A_0) = \tau$, $0 < \sigma < \infty$, $0 < \tau < \infty$, and let $\sigma(A_j) \leq \sigma$, $\tau(A_j) < \tau$ if $\sigma(A_j) = \sigma$ $(j = 0, \dots, k-1)$, then every solution $f \not\equiv 0$ of (1) satisfies $\sigma_2(f) = \sigma(A_0)$.

The authors investigate in addition the case $A_j(z) = h_j(z)\exp(P_j(z))$, where $h_j(z)$ is an entire functions; $P_j(z)$ $(j = 0, \dots, k-1)$ are polynomials with degree $n \geq 1$.

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