Guillou, Lucien

Free lines for homeomorphisms of the open annulus. (English) Zbl 1135.37016

Trans. Am. Math. Soc. 360, No. 4, 2191-2204 (2008).

In this paper the properties of the fixed point free orientation preserving homeomorphisms of the plane (so-called Brouwer homeomorphisms) are studied. Further, a Brouwer line L for the Brouwer homeomorphisms h is the image of a proper embedding of $\mathbb{R}$ into $\mathbb{R}^2$ such that L is free under h, (i.e. $h(L)\bigcap L=\emptyset$) and L separates $h(L)$ and $h^{-1}(L)$. The author proves the following result: Let H be a homeomorphism of the open annulus $S^1\times\mathbb{R}$ isotopic to the identity which admits a lift h to $\mathbb{R}^2$ without fixed point. Then there exists an essential simple closed curve in the annulus free under H (and therefore it lifts to a Brouwer line for h) or there exists a properly imbedded line in the annulus joining both ends which lifts to a Brouwer line for h. Also, the connection with the Poincaré-Birkhoff theorem is discussed.

Reviewer: Alois Klíč (Praha)

MSC:

37E30 Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces

37J40 Perturbations of finite-dimensional Hamiltonian systems, normal forms, small divisors, KAM theory, Arnol'd diffusion

Keywords:

Brouwer homeomorphism; free line; fixed point; torus; Poincaré-Birkhoff theorem

Full Text: DOI

References:

[1] Salvador Addas-Zanata, Some extensions of the Poincaré-Birkhoff theorem to the cylinder and a remark on mappings of the torus homotopic to Dehn twists, Nonlinearity 18 (2005), no. 5, 2243 – 2260. · Zbl 1084.37019 · doi:10.1088/0951-7715/18/5/018 · doi.org

[2] L. E. J. Brouwer, Beweis des ebenen Translationssatzes, Math. Ann. 72 (1912), no. 1, 37 – 54 (German). · Zbl 43.0569.02 · doi:10.1007/BF01456888 · doi.org

[3] F. Béguin, S. Crovisier, and F. Le Roux, Pseudo-rotations of the open annulus, Bull. Braz. Math. Soc. (N.S.) 37 (2006), no. 2, 275 – 306. · Zbl 1105.37029 · doi:10.1007/s00574-006-0013-2 · doi.org

[4] Mladen Bestvina and Michael Handel, An area preserving homeomorphism of \?² that is fixed point free but does not move any essential simple closed curve off itself, Ergodic Theory Dynam. Systems 12 (1992), no. 4, 673 – 676. · Zbl 0784.58037 · doi:10.1017/S014338570000701X · doi.org

[5] John Franks, Generalizations of the Poincaré-Birkhoff theorem, Ann. of Math. (2) 128 (1988), no. 1, 139 – 151. · Zbl 0676.58037 · doi:10.2307/1971464 · doi.org

[6] Lucien Guillou, Théorème de translation plane de Brouwer et généralisations du théorème de Poincaré-Birkhoff, Topology 33 (1994), no. 2, 331 – 351 (French). · Zbl 0924.55001 · doi:10.1016/0040-9383(94)90016-7 · doi.org

[7] B. de Kérékjartó, Vorlesüngen uber Topologie, Springer, Berlin (1923).

[8] B. de Kérékjartó, The plane translation theorem of Brouwer and the last geometric theorem of Poincaré, Acta Sci. Math. Szeged, 4 (1928-29), 86-102. · Zbl 54.0612.02

[9] Patrice Le Calvez, Une version feuilletée équivariante du théorème de translation de Brouwer, Publ. Math. Inst. Hautes Études Sci. 102 (2005), 1 – 98 (French, with English summary). · Zbl 1152.37015 · doi:10.1007/s10240-005-0034-1 · doi.org

[10] A. Sauzet, Application des décompositions libres à l'étude des homéomorphismes de surfaces, Ph.D. thesis, Université Paris-Nord (2001).

[11] H. Terasaka, Ein Beweis des Brouwerschen ebenen Translationssatzes, Japan J. of Math., 7 (1930), 61-69. · Zbl 56.1131.03

[12] H. E. Winkelnkemper, A generalization of the Poincaré-Birkhoff theorem, Proc. Amer. Math. Soc. 102 (1988), no. 4, 1028 – 1030. · Zbl 0656.54032 ·

[13] Gordon Whyburn and Edwin Duda, Dynamic topology, Springer-Verlag, New York-Heidelberg, 1979. Undergraduate Texts in Mathematics; With a foreword by John L. Kelley. · Zbl 0421.54001