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Representable pseudo-BCK-algebras and integral residuated lattices. (English)

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A (non-commutative) residuated lattice is an algebra $\langle L, \wedge, \vee, \cdot, \backslash, /, 1\rangle$ in which $\langle L, \vee, \wedge\rangle$ is a lattice, $\langle L, \cdot, 1\rangle$ is a monoid and $a \cdot b \leq c$ iff $b \leq a\backslash c$ iff $a \leq c/b$. A residuated lattice is called integral if 1 is the greatest element. Subalgebras of $\{\backslash, /, 1\}$-reducts of integral residuated lattices are called pseudo-BCK-algebras in this paper although they are also known as biresiduation algebras; they form a quasivariety of algebras.

The representable integral residuated lattices are those that are subdirect products of linearly ordered integral residuated lattices. The class of all integral residuated lattices forms a variety with the class of representable algebras as a proper subvariety. Axiomatizations of the variety of representable integral residuated lattices were obtained independently in [*C. J. van Alten*, J. Algebra 247, No. 2, 672–691 (2002; Zbl 1001.06012)] and [*K. Blount* and *C. Tsinakis*, Int. J. Algebra Comput. 13, No. 4, 437–461 (2003; Zbl 1048.06010)]. (The second paper also axiomatized the representable residuated lattices.) The axiomatization given in the first paper utilizes only the operations in $\{\vee, \backslash, /, 1\}$ and thus also provides an axiomatization for the class of all representable algebras within the class of subalgebras of $\{\vee, \backslash, /, 1\}$-reducts of integral residuated lattices.

An open problem that remained was to find an axiomatization that uses only the operations in $\{\backslash, /, 1\}$ and thereby also axiomatize the representable pseudo-BCK-algebras. In this paper, the author solves this very problem and shows that the identity conjectured in the first reference above does indeed axiomatize the quasivariety. The method of proof is by characterization of the relatively subdirectly irreducible members of the quasivariety of representable pseudo-BCK-algebras. This approach is different to that employed in the above-mentioned references and leads to an elegant solution of the problem. This method also serves as an alternative means of axiomatizing the variety of representable integral residuated lattices.

Reviewer: Clint van Alten (Wits)

MSC:

06F35 BCK-algebras, BCI-algebras (aspects of ordered structures)
06F05 Ordered semigroups and monoids

Cited in **12** Documents

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