Let \( \Omega^+ \) be a bounded domain in \( \mathbb{R}^3 \) and \( \Omega^- = \mathbb{R}^3 \setminus \overline{\Omega^+} \) with \( \overline{\Omega^+} = \partial \Omega^+ \cup S \), where \( S = \partial \Omega^- = \partial \Omega^+ \) is a smooth connected surface. We assume that \( \Sigma \) is a two dimensional, two-sided subsurface of some closed surface \( \Sigma_0 \) surrounding a proper subdomain \( \Omega_0 \) of \( \Omega^+ \): \( \Sigma \subset \Sigma_0 \), \( \Sigma = \Sigma \cup \partial \Sigma \), \( \Sigma_0 \subset \partial \Omega_0 \), \( \overline{\Omega_0} \subset \Omega^+ \), \( \Sigma_0 \cap S = \emptyset \). Further, let \( \Omega^c_0 = \Omega^+ \setminus \Sigma \). The following problem is studied using the integral equation method:

Find \( u, w^{(sc)} \), such that

\[
\mu \Delta u + (\lambda + \mu) \nabla \text{div} u + \rho_1 \omega^2 u = 0 \quad \text{in } \Omega^+_c, \quad \Delta w^{(sc)} + \rho_2 \omega^2 w^{(sc)} = 0 \quad \text{in } \Omega^-,
\]

\[
2[u \cdot n]^+ = b_1 [\partial_n w^{(sc)}]^+ + f_0 \quad \text{on } S, \quad [T(\partial, n)u]^+ = b_2 n[w^{(sc)}]^+ + F \quad \text{on } S,
\]

\[
[T(\partial, n)u]^+ = F^+ \quad \text{on } \Sigma, \quad [T(\partial, n)u]^+ = F^- \quad \text{on } \Sigma,
\]

\[
\partial w^{(sc)}(x)/\partial |x| - i\kappa w^{(sc)}(x) = O(|x|^{-2}) \quad \text{as } |x| \to \infty.
\]

Here \( T(\partial, n)u = 2\mu \partial_n u + \lambda n(\text{div} u) + \mu [n \times \text{curl} u] \).

Necessary and sufficient conditions for the solvability of this problem are given. It is proved that \( w^{(sc)} \) is defined uniquely, while \( u \) is defined modulo Jones mode. Some regularity results are also obtained.

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MSC:

35Q72 Other PDE from mechanics (MSC2000)
74J20 Wave scattering in solid mechanics
31A10 Integral representations, integral operators, integral equations methods in two dimensions
35J55 Systems of elliptic equations, boundary value problems (MSC2000)
74B05 Classical linear elasticity
74F10 Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)

Keywords:

transmission problems; existence; uniqueness; integral equation method; regularity

Full Text: DOI

References:


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