

**Salzmann, H.; Grundhöfer, T.; Hähl, H.; Löwen, R.**

**The classical fields. Structural features of the real and rational numbers.** (English)

Zbl 1173.00006

[Encyclopedia of Mathematics and Its Applications](#) 112. Cambridge: Cambridge University Press (ISBN 978-0-521-86516-6/hbk). xv, 401 p. (2007).

The book gives a comprehensive treatment of the classical number fields  $\mathbb{Q}$  and  $\mathbb{R}$  from almost every conceivable point of view: algebraic (additive and multiplicative groups, fields), order theoretic, topological, measure theoretic. Each of these features is studied separately, but interactions between them are very much emphasized. Typical questions are:

- Which properties characterize the real line as a topological space?
- Which properties characterize the ordered field  $\mathbb{R}$ ?
- Does the additive group of  $\mathbb{Q}$  determine the multiplicative group, and vice versa?
- What are the automorphism groups of the ordered groups of  $\mathbb{Q}$  and of  $\mathbb{R}$ ?

Several completion methods are exhibited that produce  $\mathbb{R}$  from  $\mathbb{Q}$  – non-standard numbers, order completion, topological completion. The authors include pertinent background theory about several topics, e.g., ordered groups and fields, topological groups, valuations, thus putting the theory of the classical fields in a larger context. The closely related number systems of non-standard numbers and  $p$ -adic numbers are included, together with some of their basic properties.

Reviewer: [Niels Schwartz \(Passau\)](#)

**MSC:**

- 00A05** Mathematics in general
- 12-01** Introductory exposition (textbooks, tutorial papers, etc.) pertaining to field theory
- 54-01** Introductory exposition (textbooks, tutorial papers, etc.) pertaining to general topology
- 22-01** Introductory exposition (textbooks, tutorial papers, etc.) pertaining to topological groups
- 03-01** Introductory exposition (textbooks, tutorial papers, etc.) pertaining to mathematical logic and foundations

Cited in **13** Documents

**Keywords:**

real numbers; rational numbers;  $p$ -adic numbers; non-standard numbers; ordered group; ordered field; topological space; topological group; topological field