McMillan, T.; **Holmes, P.**
An elastic rod model for anguilliform swimming. (English) Zbl 1113.92005
J. Math. Biol. 53, No. 5, 843-886 (2006).

Summary: We develop a model for anguilliform (eel-like) swimming as an elastic rod actuated via time-dependent intrinsic curvature and subject to hydrodynamic drag forces, the latter as proposed by *G. Taylor* [Proc. R. Soc. Lond., Ser. A 214, 158–183 (1952; Zbl 0047.43901)]. We employ a geometrically exact theory and discretize the resulting nonlinear partial differential evolution both to perform numerical simulations, and to compare with previous models consisting of chains of rigid links or masses connected by springs, dampers, and prescribed force generators representing muscles. We show that muscle activations driven by motoneuronal spike trains via calcium dynamics produce intrinsic curvatures corresponding to near-sinusoidal body shapes in longitudinally-uniform rods, but that passive elasticity causes Taylor's assumption of prescribed shape to fail, leading to time-periodic motions and lower speeds than those predicted by Taylor (loc. cit.).

We investigate the effects of bending stiffness, body geometry, and activation patterns on swimming speed, turning behavior, and acceleration to steady swimming. We show that laterally-uniform activation yields stable straight swimming and laterally differential activation levels lead to stable turns, and we argue that tapered bodies with reduced caudal (tail-end) activation (to produce uniform intrinsic curvature) swim faster than ones with uniform activation.

MSC:

92C10 Biomechanics
74A99 Generalities, axiomatics, foundations of continuum mechanics of solids
92C20 Neural biology

Full Text: DOI

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