Walther, Uli

Duality and monodromy reducibility of A-hypergeometric systems. (English) Zbl 1126.33006

Math. Ann. 338, No. 1, 55-74 (2007).

The author studies two behaviors of a hypergeometric system. Let A be an integer-entries $d \times n$ matrix, $\beta \in \mathbb{C}^d$ a parameter and $H_A(\beta)$ the A-hypergeometric system (or GKZ-system) [*I. M. Gel'fand, I. M. Graev* and *A. V. Zelevinskii*, Dokl. Akad. Nauk SSSR 295, 14–19 (1987; Zbl 0661.22005)]. First he studies whether $H_A(\beta)$ has irreducible monodromy, that is, $H_A(\beta) \bigotimes_{\mathbb{C}[x]} \mathbb{C}(x)$ is irreducible as a $\mathbb{C}(x)$-module. He proves that $H_A(\beta)$ has irreducible monodromy for almost all $\beta \in \mathbb{C}^d$, that is, except for a proper Zariski closed subset of $\mathbb{C}^d$. In the proof, the notion of rank-jumping plays a key role. It was introduced in [*L. F. Matusevich, E. Miller* and *U. Walther*, J. Am. Math. Soc. 18, No. 4, 919–941 (2005; Zbl 1095.13033)]. Next he studies the holonomic dual of $H_A(\beta)$. He proves that it is a GKZ-system for almost all β. He also studies the structure of the exceptional subset.

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MSC:

33C60 Hypergeometric integrals and functions defined by them (E, G, H and I functions)

33C80 Connections of hypergeometric functions with groups and algebras, and related topics

Keywords:

GKZ-system; holonomic D-module; rank-jumping

Full Text: DOI arXiv

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